Applying interval PCA and clustering to quantile estimates: empirical distributions of fertilizer cost for yearly crops in European Countries

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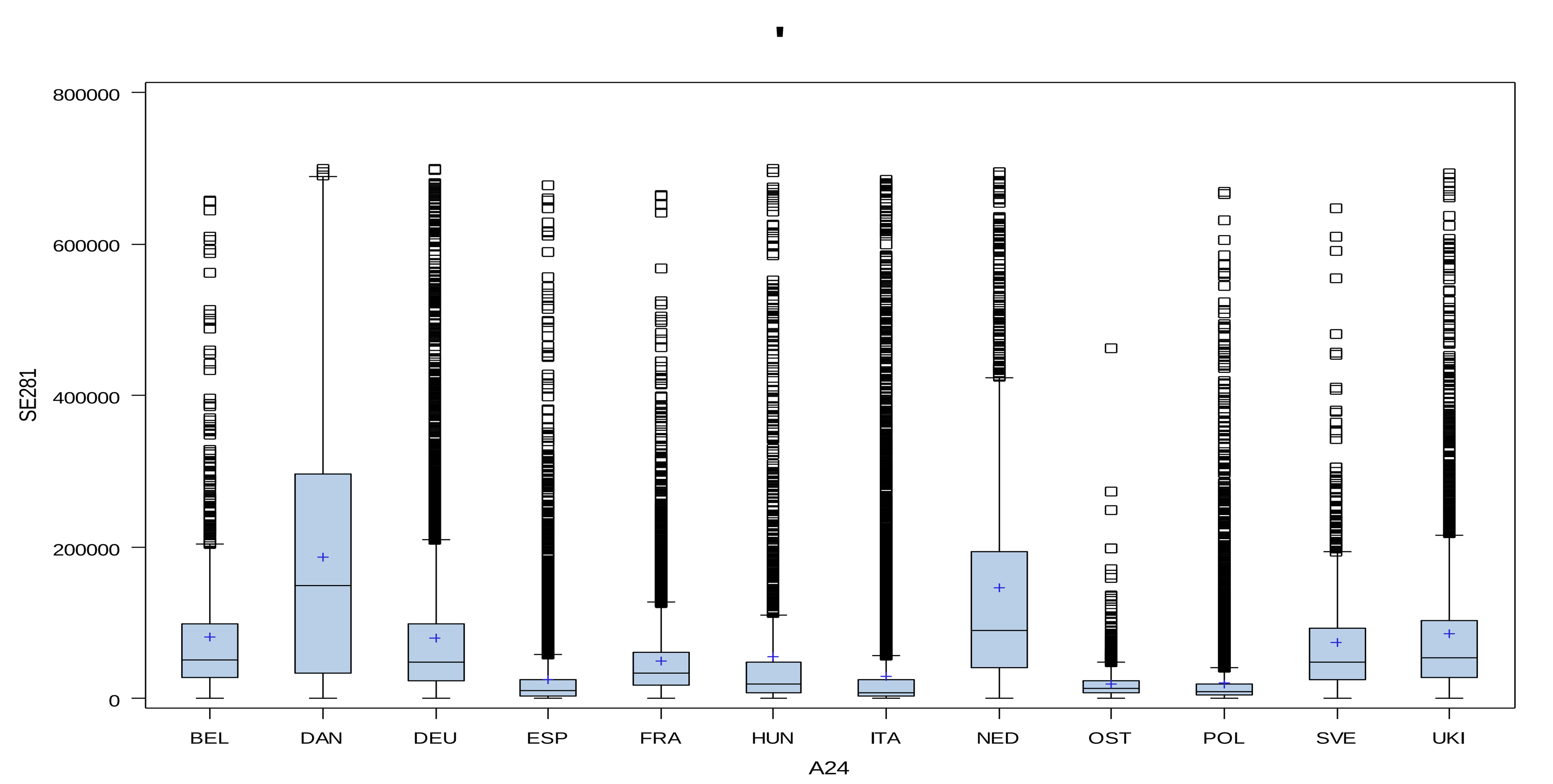
After recalling the conceptual framework of the estimation of agricultural production costs, we present the empirical data model, the quantile regression approach and the interval principal component analysis and clustering tools used to obtain typologies of European countries on the basis of the conditional quantile distributions of fertilizer cost empirical estimates issued from FADN data.

1. Distribution of Production Costs in Agriculture



2. Input Allocation Model

$$X_{ih} = \sum_{k=1}^{K} \alpha_{ih}^{k} Y_{kh} + \varepsilon_{ih}$$

Charges \ Produits	Y_{1h}	...	Y_{kh}	...	Y_{Kh}	Total charge
X_{1h}	a_{1h}^1	⋯	a_{1h}^k	⋯	a_{1h}^K	$\sum X_{1h}$
⋮	⋮		⋮		⋮	⋮
X_{ih}	a_{ih}^1	⋯	a_{ih}^k	⋯	a_{ih}^K	$\sum X_{ih}$
⋮	⋮		⋮		⋮	⋮
X_{Ih}	a_{Ih}^1	...	a_{Ih}^k	⋯	a_{Ih}^K	$\sum X_{Ih}$
Total produit	$\sum Y_{1h}$	⋯	$\sum Y_{kh}$	⋯	$\sum Y_{Kh}$	$\sum_k Y_{kh} = \sum_i X_{ih}$

3. Estimation Method: Quantile Regression (Koenker, 2005)

$$\hat{\beta}_{\omega}(q) = \arg\min_{\beta \in \Re^p} \left\{ \sum_{i \in \{i / x_i \geq y_i'\beta\}} \omega_i q \left| x_i - y_i'\beta \right| + \sum_{i \in \{i / x_i \leq y_i'\beta\}} \omega_i (1-q) \left| x_i - y_i'\beta \right| \right\}$$

4. Estimation Intervals, Yield Crops (EU12, 2006)

Unit Costs for € 1 of gross product

Country	D1	Q1	Q2	Q3	D9
Austria	[0.000 ; 0.029]	[0.043 ; 0.057]	[0.068 ; 0.086]	[0.106 ; 0.127]	[0.155 ; 0.179]
Belgium	[0.009 ; 0.019]	[0.023 ; 0.030]	[0,038 ; 0.047]	[0.056 ; 0.080]	[0.082 ; 0.110]
Denmark	[0.018 ; 0.024]	[0.035 ; 0.035]	[0.056 ; 0.056]	[0.094 ; 0.094]	[0.140 ; 0.140]
France	[0.023 ; 0.028]	[0.053 ; 0.065]	[0.125 ; 0.125]	[0.182 ; 0.182]	[0.232 ; 0.232]
Germany	[0.004 ; 0.009]	[0.025 ; 0.033]	[0.082 ; 0.082]	[0.140 ; 0.140]	[0.181 ; 0.181]
Hungary	[0.020 ; 0.038]	[0.056 ; 0.071]	[0.093 ; 0.110]	[0.138 ; 0.164]	[0.197 ; 0.197]
Italy	[0.007 ; 0.011]	[0.019 ; 0.022]	[0.041 ; 0.041]	[0.078 ; 0.078]	[0.121 ; 0.121]
Netherlands	[0.001 ; 0.004]	[0.004 ; 0.006]	[0.009 ; 0.012]	[0.017 ; 0.022]	[0.026 ; 0.029]
Poland	[0.024 ; 0.032]	[0.052 ; 0.059]	[0.088 ; 0.099]	[0.146 ; 0.165]	[0.215 ; 0.228]
Spain	[0.013 ; 0.017]	[0.025 ; 0.033]	[0.058 ; 0.058]	[0.103 ; 0.103]	[0.169 ; 0.169]
Sweden	[-0.007 ; 0.016]	[0.003 ; 0.038]	[0.100 ; 0.100]	[0.215 ; 0.215]	[0.293 ; 0.293]
United-Kingdom	[0.006 ; 0.029]	[0.036 ; 0.047]	[0.088 ; 0.088]	[0.137 ; 0.137]	[0.171 ; 0.171]

5. Distribution Analysis of Interval Estimates

$$z_l^q = \left[Inf\_\hat{\gamma}_l^{j_0}(q) ; Sup\_\hat{\gamma}_l^{j_0}(q) \right] = \left[\underline{z_l^q} ; \overline{z_l^q} \right]$$

PCA of Estimation Intervals in Mixed Strategy

$$\hat{Z}' P_{\Phi} \hat{Z} = Z' A (A'A)^{-1/2} P_{\Phi} (A'A)^{-1/2} A' Z s_m = \rho_m s_m$$

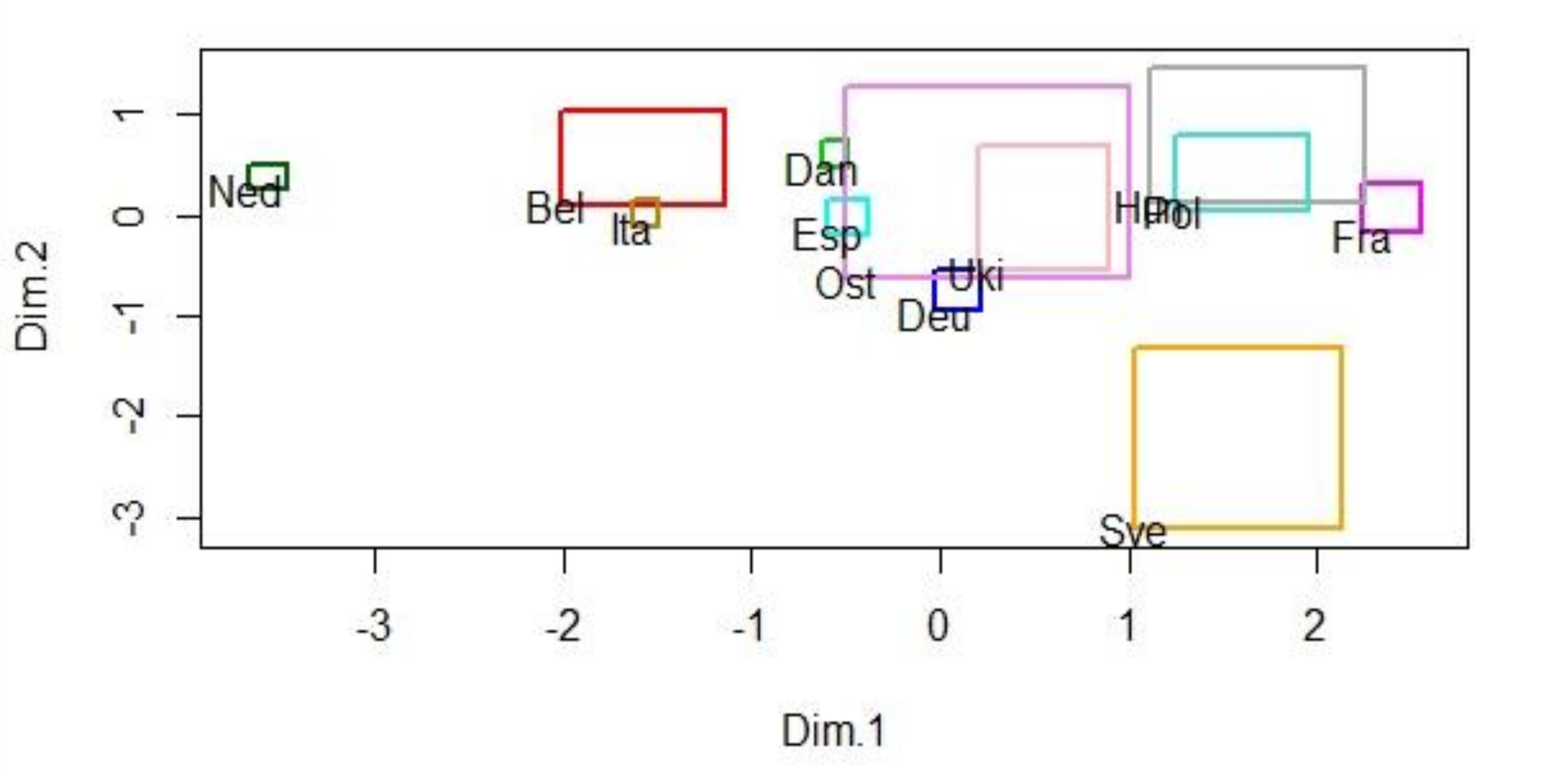
(Lauro & Palumbo, 2000)

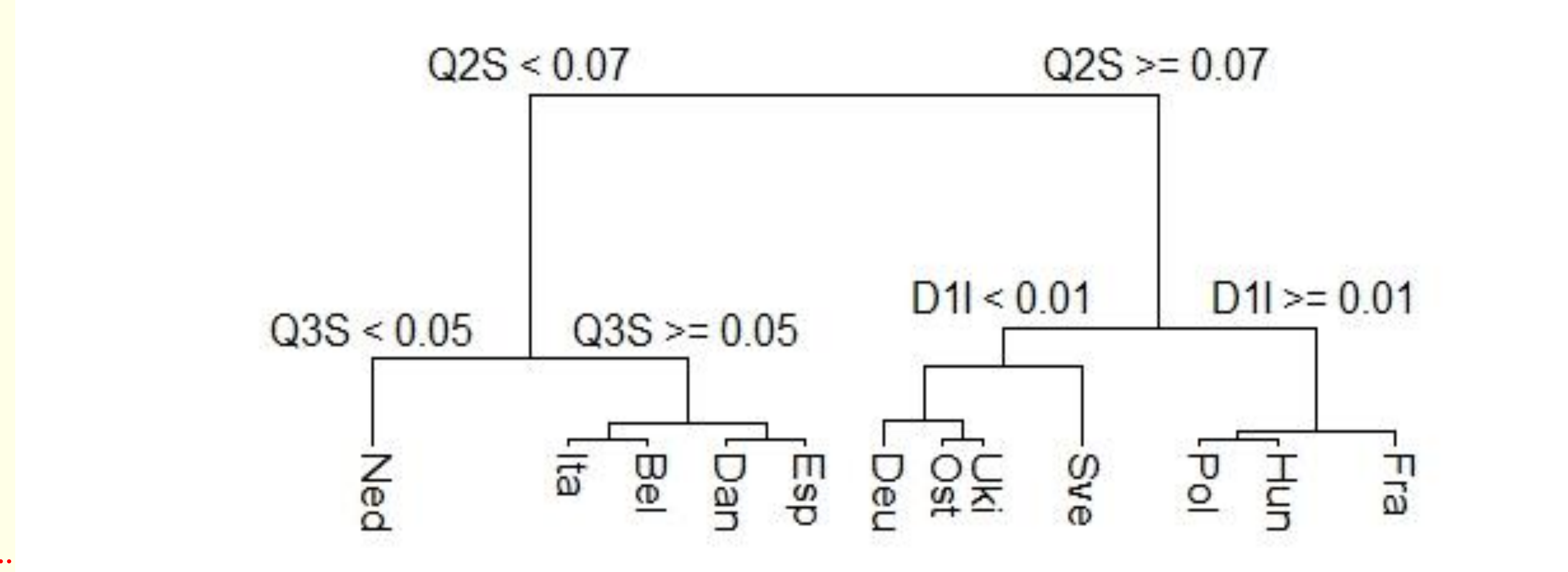
Hierarchical Divisive Classification on Interval Data

$$d(\omega_l, \omega_{l'}) = \left(\sum_{q=1}^{Q} \delta_M^2 \left(z_l^q, z_{l'}^q \right) \right)^{1/2}$$

(Fuentes & Chavent, 2015)

6. Factor Plane (Interval PCA) and Divisive Segmentation Tree





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