# *Quantum Many-Body Physics*

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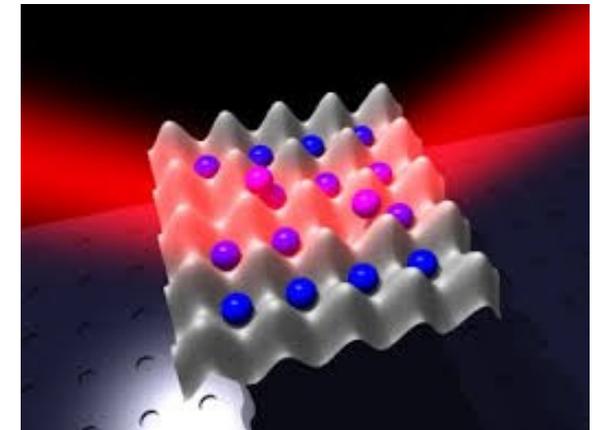https://www.cpht.polytechnique.fr/cpht/uquantmat/

## Research field

Theory of quantum many-body systems

Quantum science and technologies ; Quantum simulation with ultracold atoms

Understand the behavior of correlated quantum matter and how to simulate them with controlled quantum systems

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# Quantum Many-Body Physics

1. Introduction to many-body quantum physics

  Few-body versus many-body problems

  Quantum versus classical many-body systems

  Quantum phases and quantum phase transitions

2. Models and methods

  Second quantization approach

  How to build a many-body model (a heuristic approach)

  Case study of the Bose-Hubbard model

3. Strongly-correlated bosons in shallow quasiperiodic potentials

  Physics of disorder

  Bose-glass physics in shallow quasiperiodic potentials

  Quantum quasicrystals : The Bose side

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# From Few-Body Physics …

Understanding the physics of systems made of constituents that interact with each other

Many applications at all scales

A difficult problem already for 3 particles (eg dynamical instabilities, chaos, …)
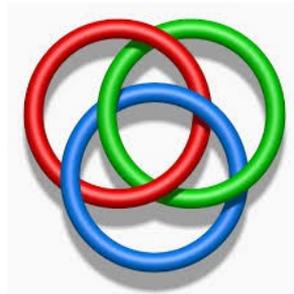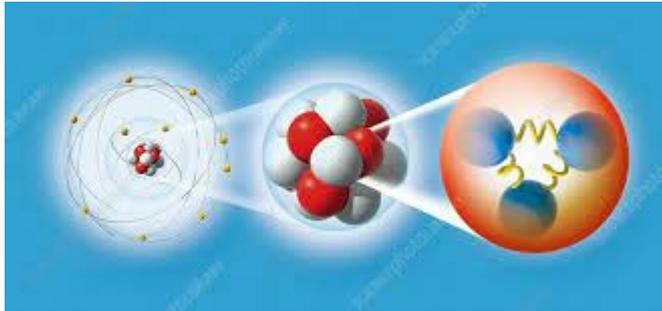
# … to Many-Body Physics

Systems made of huge number of constituents, which all interact with each other

$10^{4\text{-}6}$ (ultracold atoms), $10^{23}$ (Avogadro), $10^{27}$ (human body), $10^{50}$ (Earth), $10^{80}$ (Universe)

Chaos and statistical physics
(benefits of complexity)

Linear response theory

Linearity of quantum physics
and the problem of
thermalization (ergodicity,
eigenstate thermalization
hypothesis, and many-body
localization)
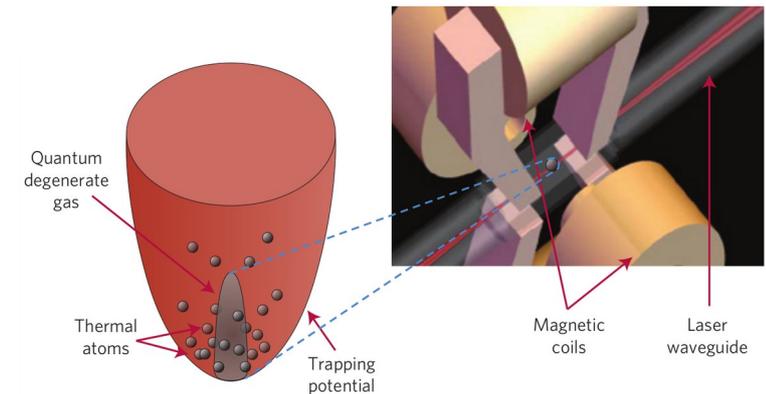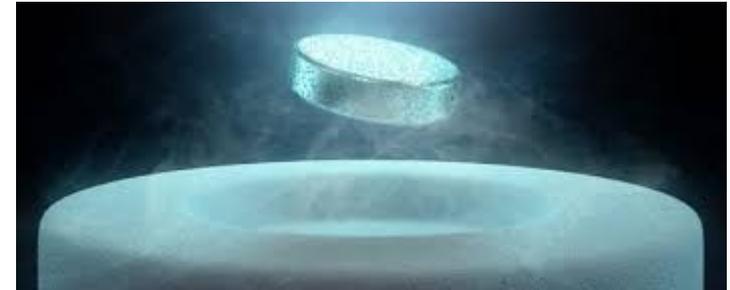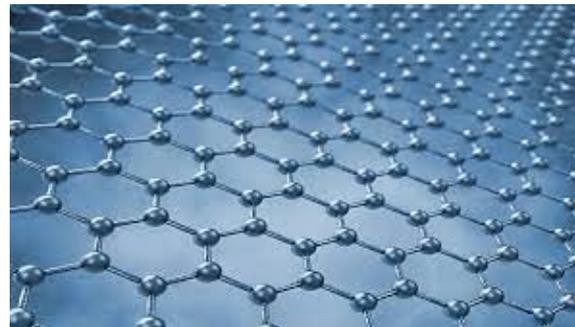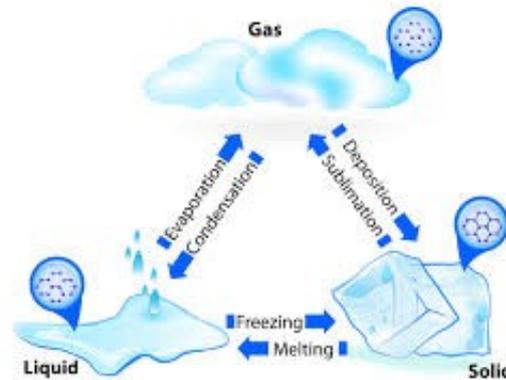
# Quantum Many-Body Systems

## In spite of linearity, major issues arise

Quantum coherence (interference), non-commuting observables

Few-body interactions but collective effects, including non-separability of constituents

Entanglement, huge Hilbert space

Quantum indiscernability (bosons, fermions, symmetrization and anti-symmetrization) and spins
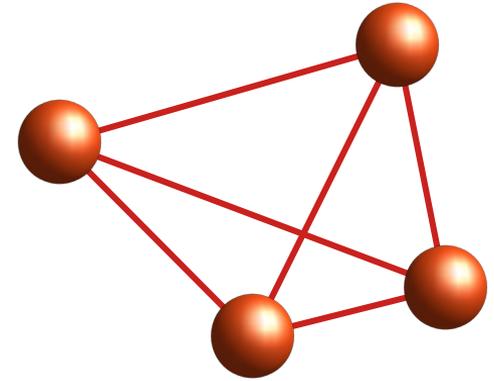
## Theoretical physics has developed a number of tools, yet

Many-body physics remains a major issue in condensed matter physics and beyond

Each system requires specific attention and educated approximations

# Phase Transitions and Emerging Phenomena

## Phase transitions in classical many-body systems

Distinctive characteristics of a many-body system (« More is different », P. Anderson, 1972)

Solid-liquid-gas transitions ; Magnetism, etc …

Classical phase transitions driven by thermal fluctuations
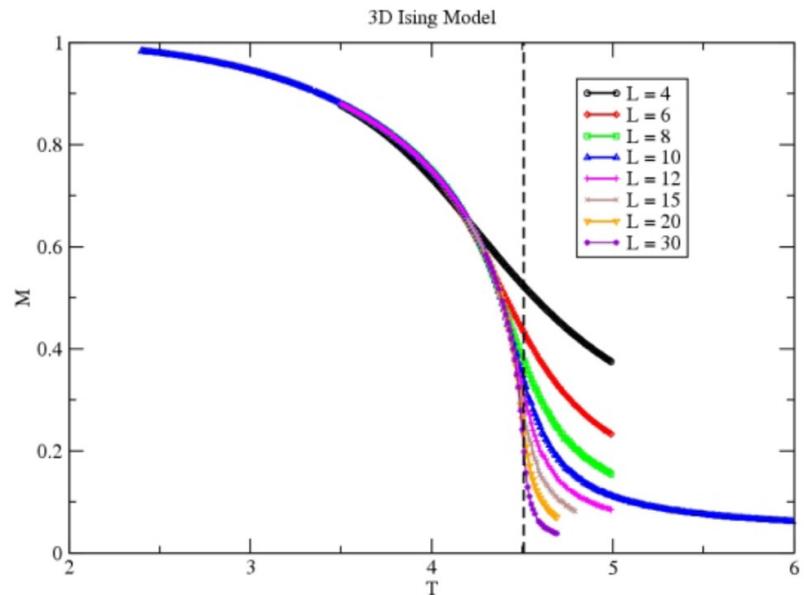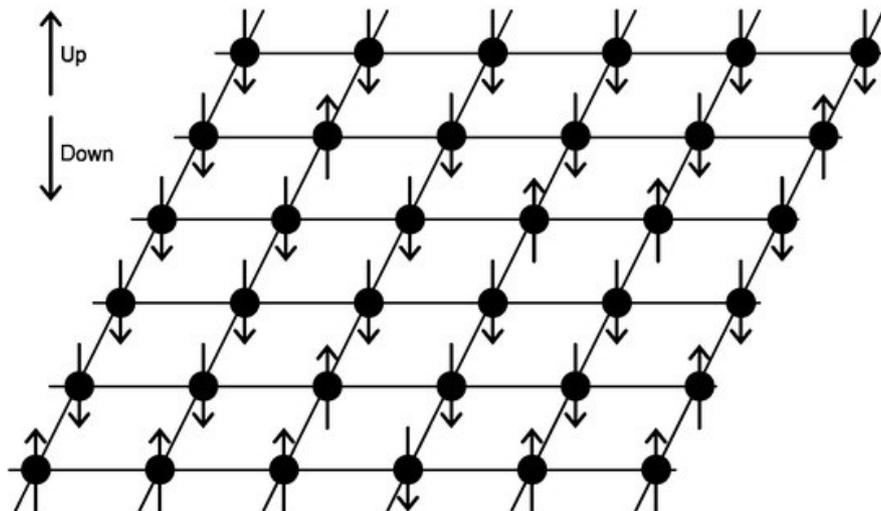
# Phase Transitions and Emerging Phenomena

## Phase transitions in quantum many-body systems

Phase transitions driven by thermal fluctuations (*classical phase transitions* ; superconductivity, superfluidity, magnetic transitions, …)

Phase transitions driven by quantum fluctuations (*quantum phase transitions* ; metal-insulator transitions, quantum magnetic transitions, …)

Transverse-field Ising chain
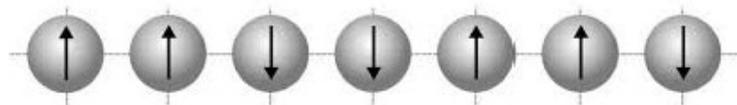


$$\hat{H} = -J \sum_{\langle j, \ell \rangle} \hat{\sigma}_j^x \cdot \hat{\sigma}_\ell^x + h \sum_j \hat{\sigma}_j^z$$

$$g = h/J$$
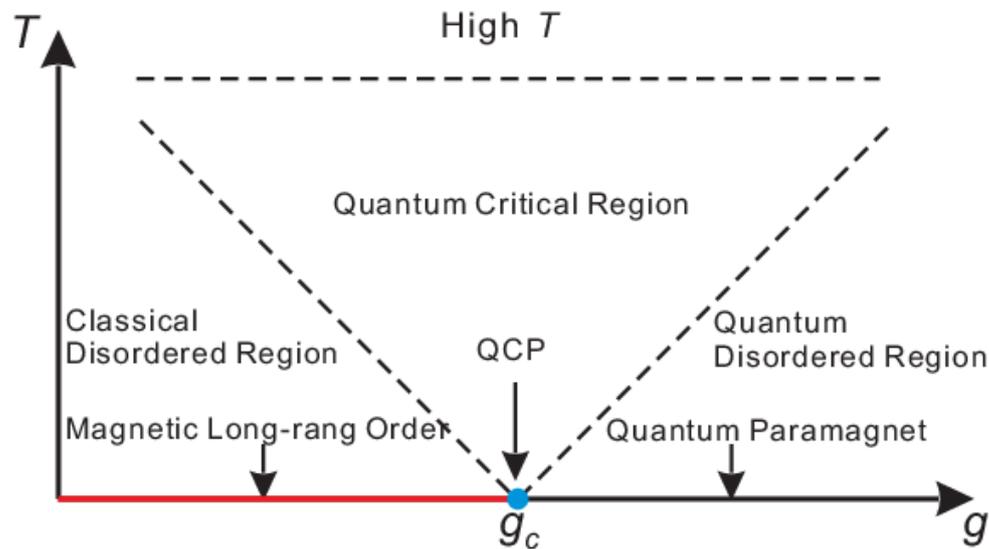
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# Second Quantization Formalism

A new formalism adapted to indistinguishable quantum particles

**Bosons** $(\eta = +1; \, n_\lambda \in \mathbb{N})$

$$\hat{a}_\lambda |n_1...n_\lambda...\rangle \equiv \sqrt{n_\lambda} \, |n_1...n_\lambda - 1...\rangle$$

$$\hat{a}_\lambda^\dagger |n_1...n_\lambda...\rangle \equiv \sqrt{n_\lambda + 1} \, |n_1...n_\lambda + 1...\rangle$$

$$\hat{n}_\lambda = \hat{a}_\lambda^\dagger \hat{a}_\lambda$$

$$[\hat{a}_\lambda, \hat{a}_{\lambda'}] = [\hat{a}_\lambda^\dagger, \hat{a}_{\lambda'}^\dagger] = 0$$

$$[\hat{a}_\lambda, \hat{a}_{\lambda'}^\dagger] = \delta_{\lambda,\lambda'}$$

**Fermions** $(\eta = -1; \, n_\lambda \in \{0, 1\})$

$$\hat{a}_\lambda |n_1...n_\lambda...\rangle \equiv \prod_{\lambda' < \lambda} (-1)^{n_{\lambda'}} \, |n_1...n_\lambda - 1...\rangle$$

$$\hat{a}_\lambda^\dagger |n_1...n_\lambda...\rangle \equiv \prod_{\lambda' < \lambda} (-1)^{n_{\lambda'}} \, |n_1...n_\lambda + 1...\rangle$$

$$\hat{n}_\lambda = \hat{a}_\lambda^\dagger \hat{a}_\lambda$$

$$\{\hat{a}_\lambda, \hat{a}_{\lambda'}\} = \{\hat{a}_\lambda^\dagger, \hat{a}_{\lambda'}^\dagger\} = 0$$

$$\{\hat{a}_\lambda, \hat{a}_{\lambda'}^\dagger\} = \delta_{\lambda,\lambda'}$$

## One- and two-body observables

First quantization $\longrightarrow$ Second quantization

$$\hat{\mathcal{A}} = \sum_{1 \leq j \leq N} \hat{A}^{(j)} \qquad \hat{\mathcal{A}} = \sum_{\lambda,\lambda'} \langle \lambda' | \hat{A} | \lambda \rangle \, \hat{a}_{\lambda'}^\dagger \hat{a}_\lambda$$

$$\hat{\mathcal{B}} = \frac{1}{2} \sum_{j \neq \ell} \hat{B}^{(j,\ell)} \qquad \hat{\mathcal{B}} = \frac{1}{2} \sum_{\lambda_1,\lambda_2,\lambda_1',\lambda_2'} \langle \lambda_1', \lambda_2' | \hat{B} | \lambda_1, \lambda_2 \rangle \, \hat{a}_{\lambda_1'}^\dagger \hat{a}_{\lambda_2'}^\dagger \hat{a}_{\lambda_2} \hat{a}_{\lambda_1}$$

H. Bruus & K. Flensberg, *Many-body Quantum Theory in Condensed Matter Physics* (Oxford Univ Press, UK, 2004)

J. W. Negele & H. Orland, *Quantum many-particle systems* (Perseus books, USA, 1998)

# Quantum Many-Body Physics

# Physics of Disorder : From Condensed Matter to Ultracold Atoms
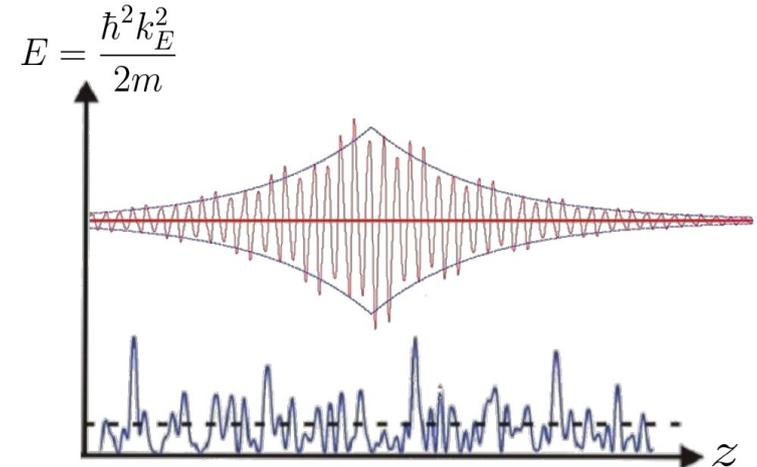
## Disorder is ubiquitous in condensed matter

Almost any real-life material is subjected to defects

Disorder may strongly affects the physics of the material
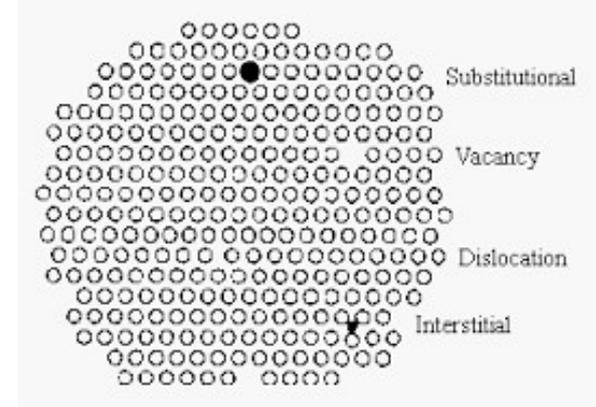
→ Localization of single-particle wavefunctions (Anderson, 1958)

→ Wavefunctions are always localized in 1D and 2D ; Localization transition in 3D (Anderson *et al.*, 1979)
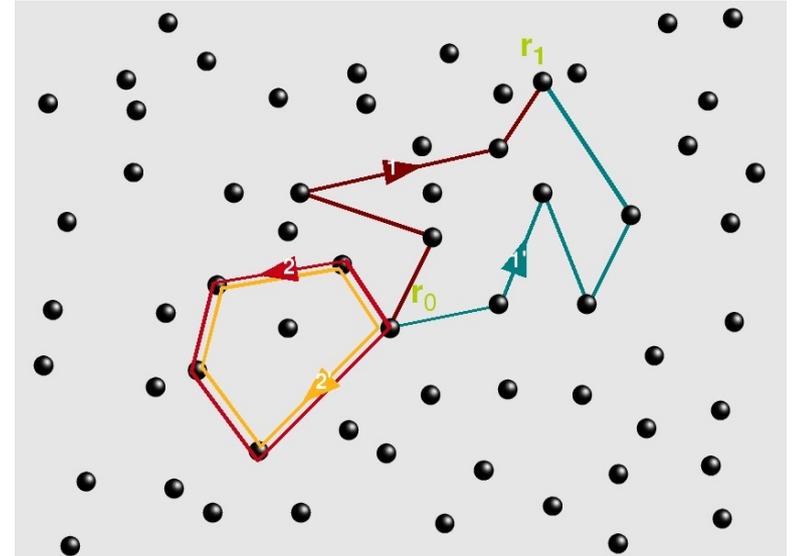


$$E = \frac{\hbar^2 k_E^2}{2m}$$

# Physics of Disorder : From Condensed Matter to Ultracold Atoms

## Wave physics and disorder

May be interpreted as an interference effect of waves scattering from the impurities

→ Single-particle effect : Difficult to observe in traditional condensed matter systems

→ May be observed with optical waves?

→ Many attempts all refuted up to mid-2000's



## What about correlated quantum matter?

Very important (and difficult) problem in condensed matter (interplay of coherence, interactions, and disorder)

# Ultracold Atoms
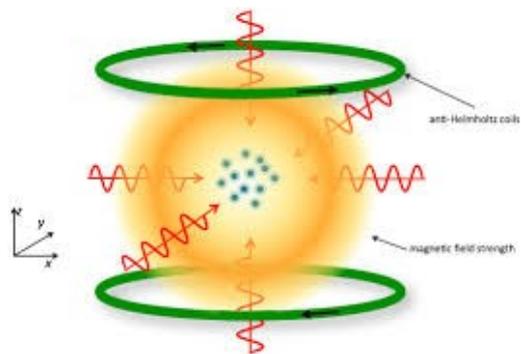


Laser cooling (~40-100μK)

Optical trapping
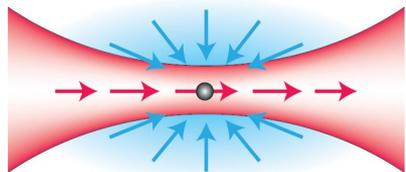


Evaporative cooling (<100nK)

➔ $\lambda_{dB} \gtrsim 500$ nm $\sim \lambda_{opt}$

➔ $k_B T \lesssim E_r$
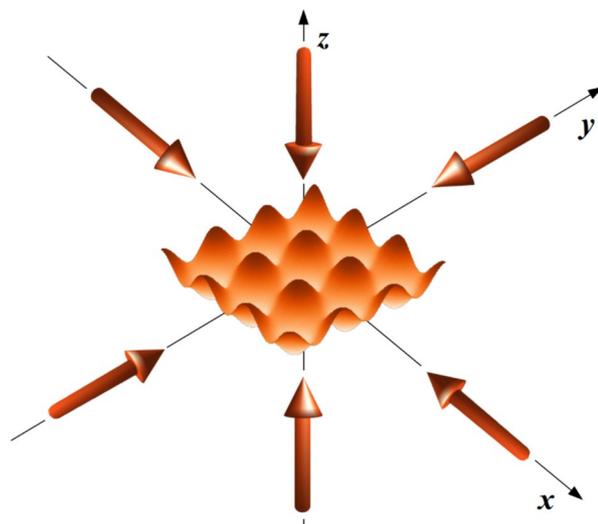
Optical design
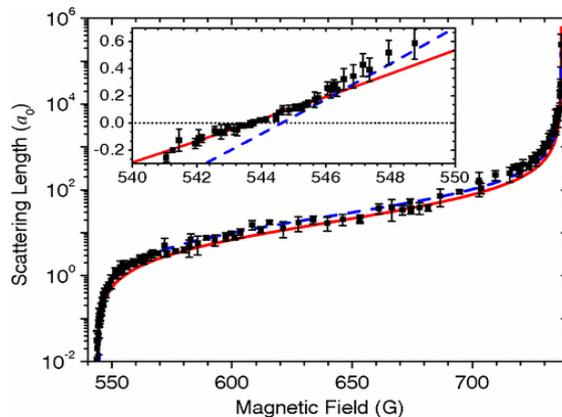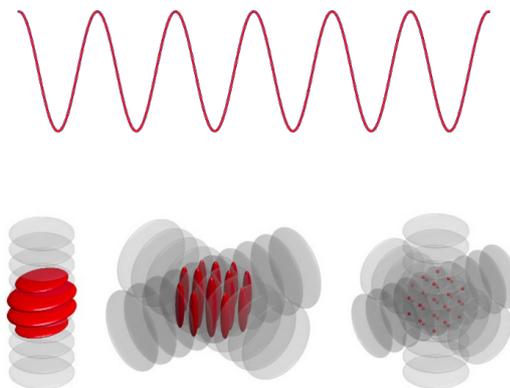
Control of interactions

→ A wonderful playground
   for basic science
   (quantum simulation)

# Disordered Quantum Gases Under Control

Early proposals :
- Grynberg (ENS Paris, 2000) : Classical particles in disorder
- Lewenstein (Hannover, 2003) : Strongly-correlated quantum gases in disorder
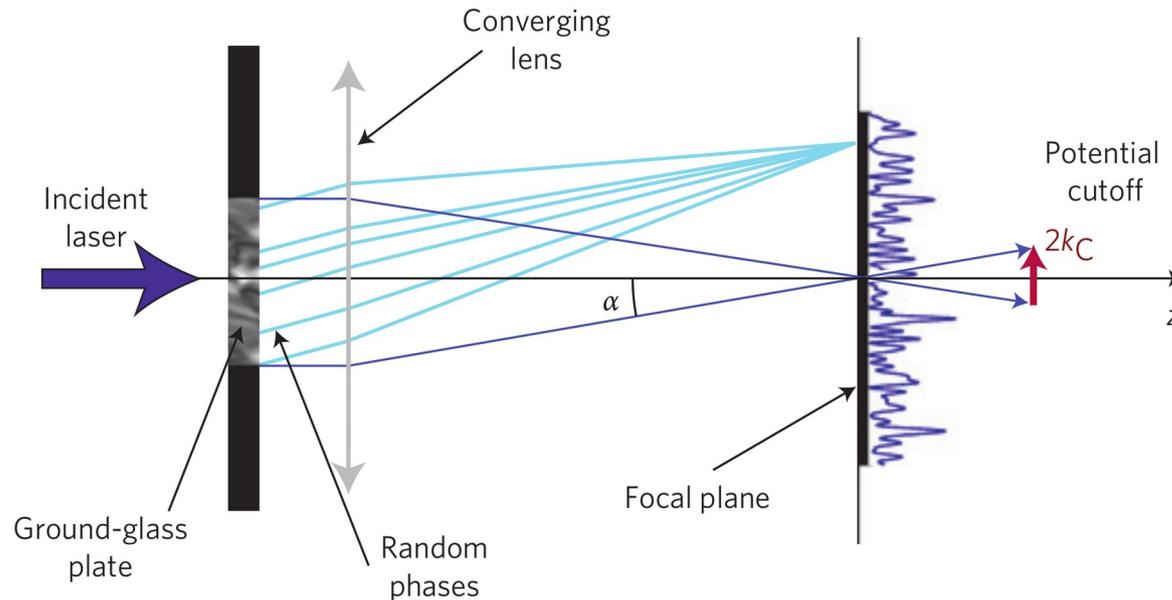


Controlled amplitude and correlation length/function

# Disordered Quantum Gases Under Control



Exponential Anderson localization may be directly observed in expanding condensates and the localization length can be computed exactly !

Sanchez-Palencia *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
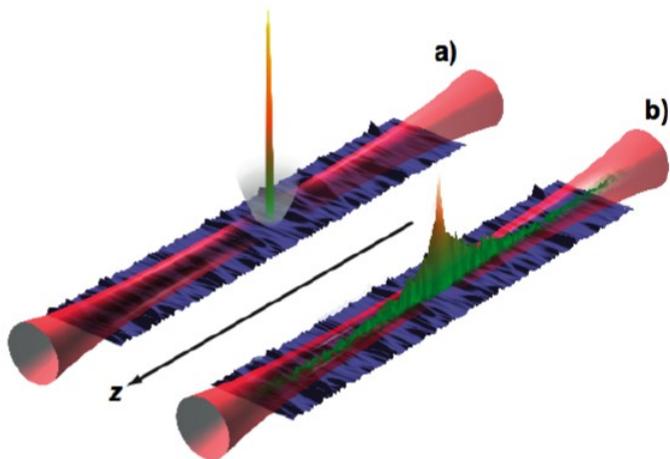
Aspect, Bouyer, Josse, Sanchez-Palencia
[Nature **453**, 891 (2008)]

→ Agreement without any fitting parameter



$$L_{loc} = \frac{2\hbar^4 k^{\max 2} k_c}{\pi m^2 V_R^2 (1 - k^{\max}/k_c)}$$

L. Sanchez-Palencia et al.,
PRL **98**, *210401*, 2007
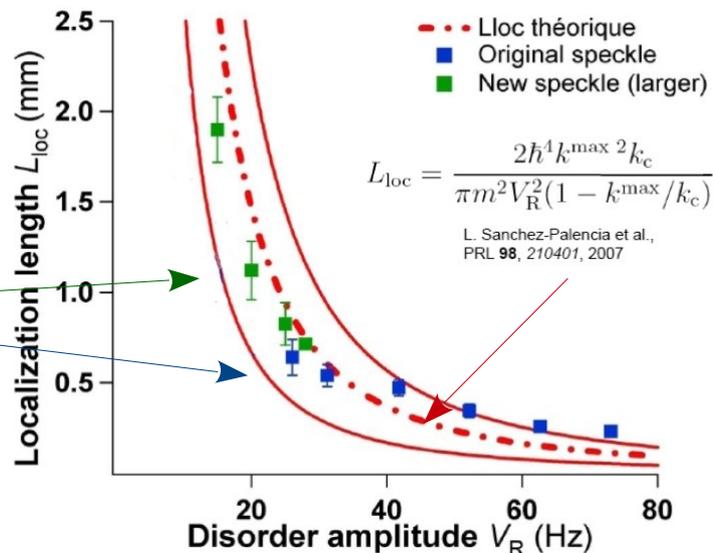
Progress paper : Sanchez-Palencia & Lewenstein, Nat. Phys. **6**, 87 (2010)

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# Localization in the Aubry-André Model
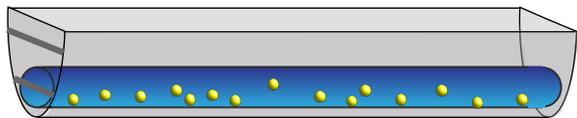


→ 1D tubes created by a very strong 2D lattice
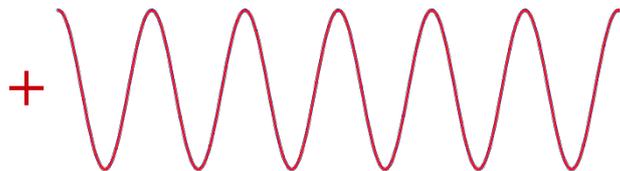
→ Tight-binding model created by a strong 1D lattice

→ Quasi-periodic modulation created by a weak 1D lattice with incommensurate period



## Observation of localization in the Aubry-André model

$$\hat{H} = -J \sum_j \left( \hat{a}^\dagger_{j+1} \hat{a}_j + \text{H.c.} \right) + \sum_j \Delta \cos\left( 2\pi\beta j + \varphi \right) \hat{n}_j$$

with $\beta \in \mathbb{Q}^c$

Inguscio, Modugno [Nature **453**, 895 (2008)]
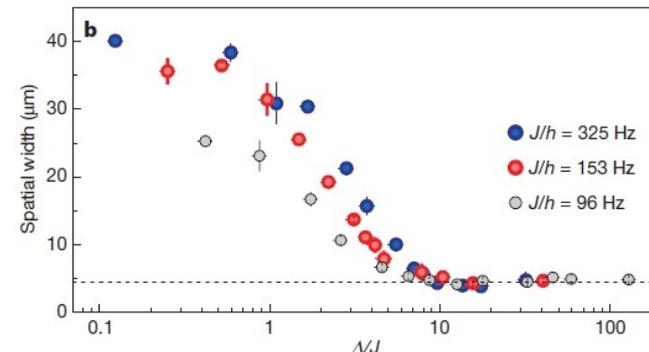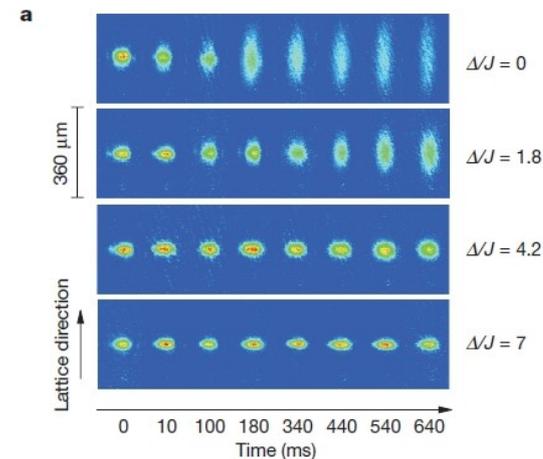
# Bose-Glass Physics in the Hubbard-Aubry-André Model

# Bose-Glass Physics in the Hubbard-Aubry-André Model



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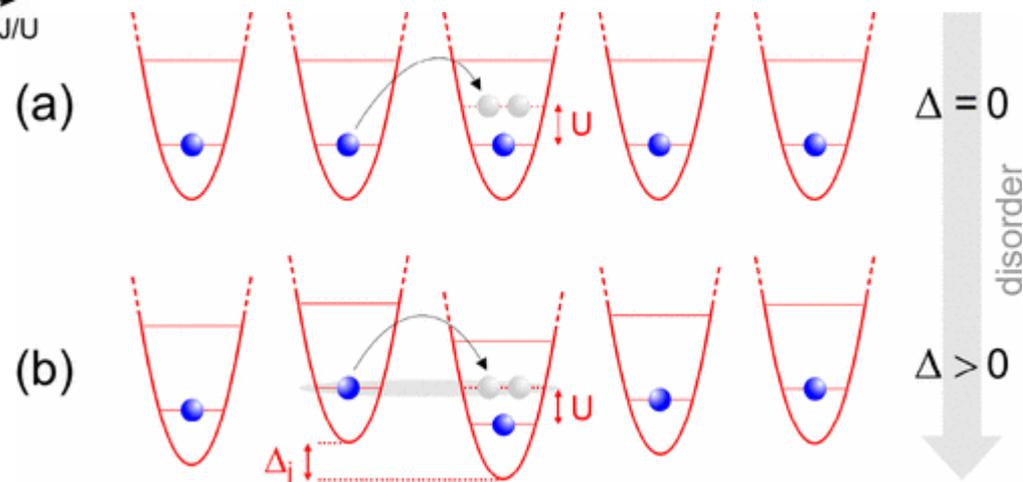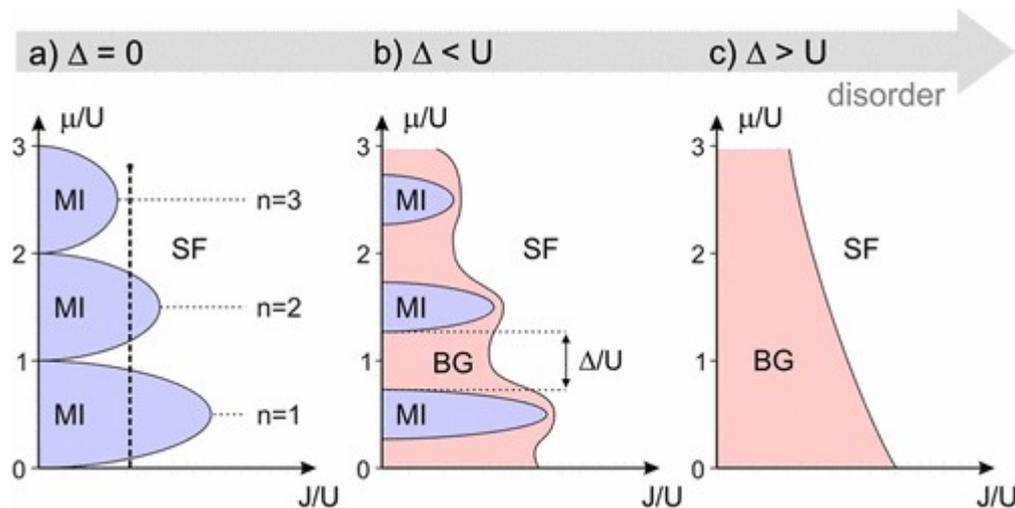


experiment

DMRG+heuristic
thermal broadening

DMRG

Inguscio, Modugno, Giamarchi, Roux [Phys. Rev. Lett.**113**, 095301 (2014)]

Mobility and coherence measurements
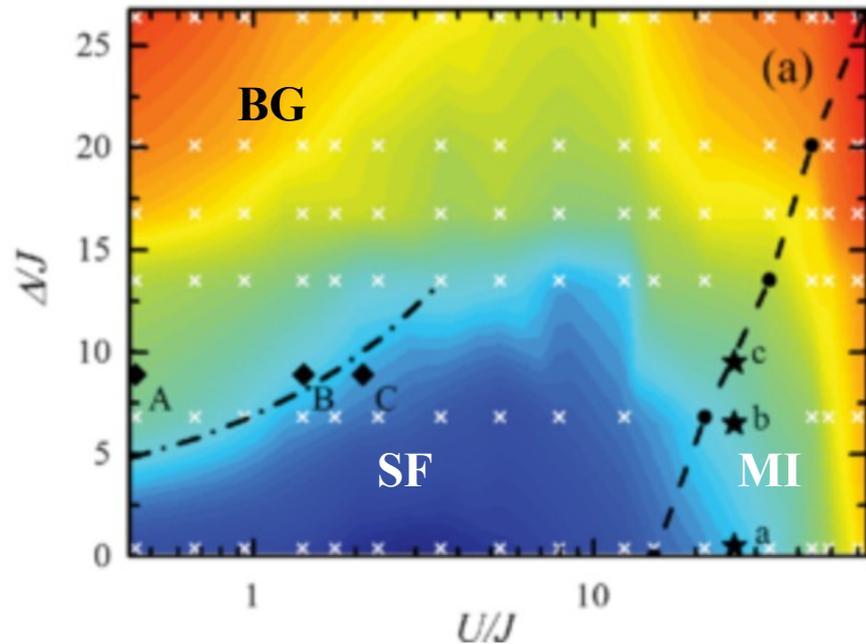


→ Coherence blurred by thermal fluctuations

$$J, k_B T \ll E_r \quad \text{but} \quad J \sim k_B T$$

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# Localization in a Shallow Quasiperiodic Lattice

Temperature is hard to further decrease
→ Scale up characteristic energies !

Do particles localize in a shallow quasiperiodic potential?
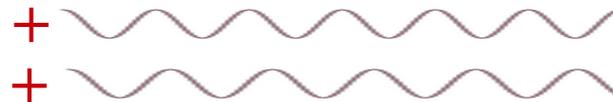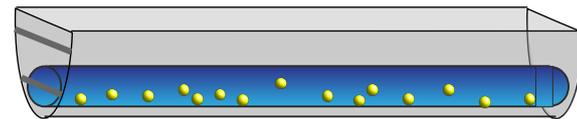
Inverse participation ratio

$$\text{IPR} = \int dx\, |\psi(x)|^4$$

critical potential
$V_c \simeq 1.112\, E_r$

IPR $\sim L^{-1}$ → extended

mobility edge

IPR $\sim L^0$ → localized
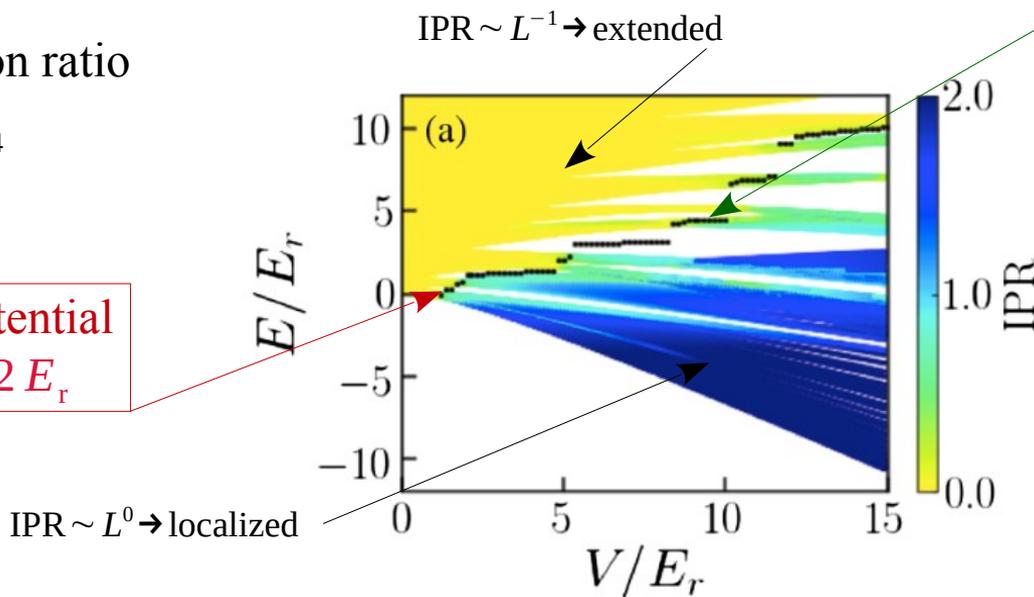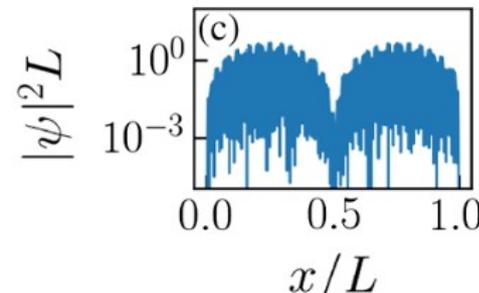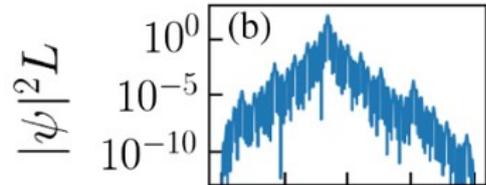
# Lieb-Liniger Gas in a Shallow Quasiperiodic Lattice

Let now add inter-particle interactions

$$\mathcal{H} = \sum_{1 \leq j \leq N} \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_j^2} + V(x_j) \right] + g \sum_{j < \ell} \delta(x_j - x_\ell)$$

Quantum Monte Carlo in continuous space
(compressibility and superfluid fraction)
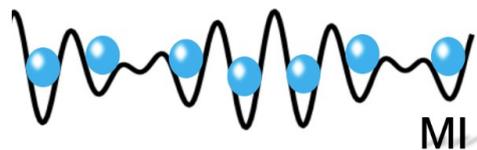
Incompressible
Non superfluid

**MI**
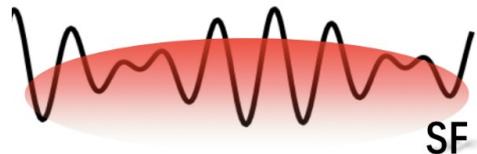
(Compressible)
Superfluid

**SF**

Compressible
Non superfluid

**BG**

$V = 1.5 \, E_r$



H. Yao , T. Giamarchi & L. Sanchez-Palencia, Phys. Rev. Lett. **125**, 060401 (2020)
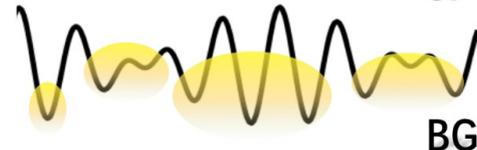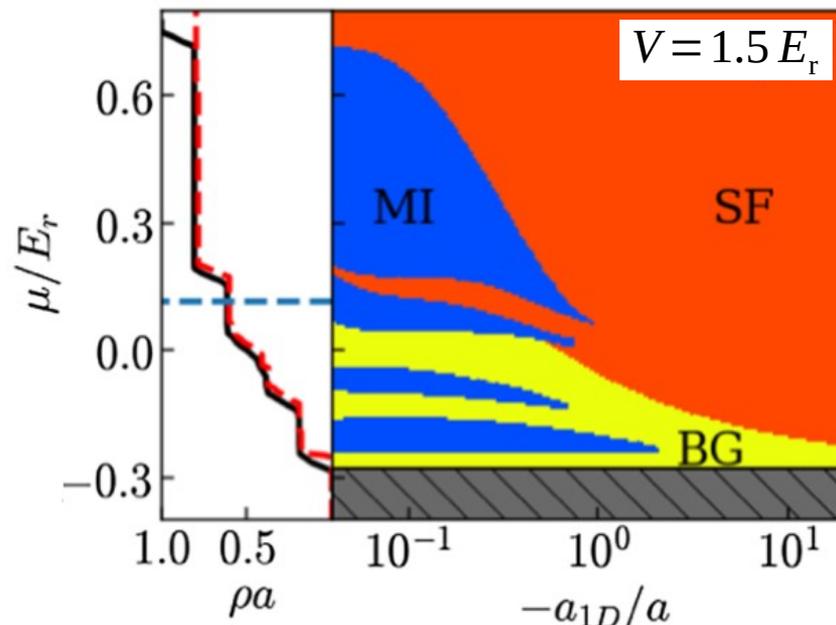
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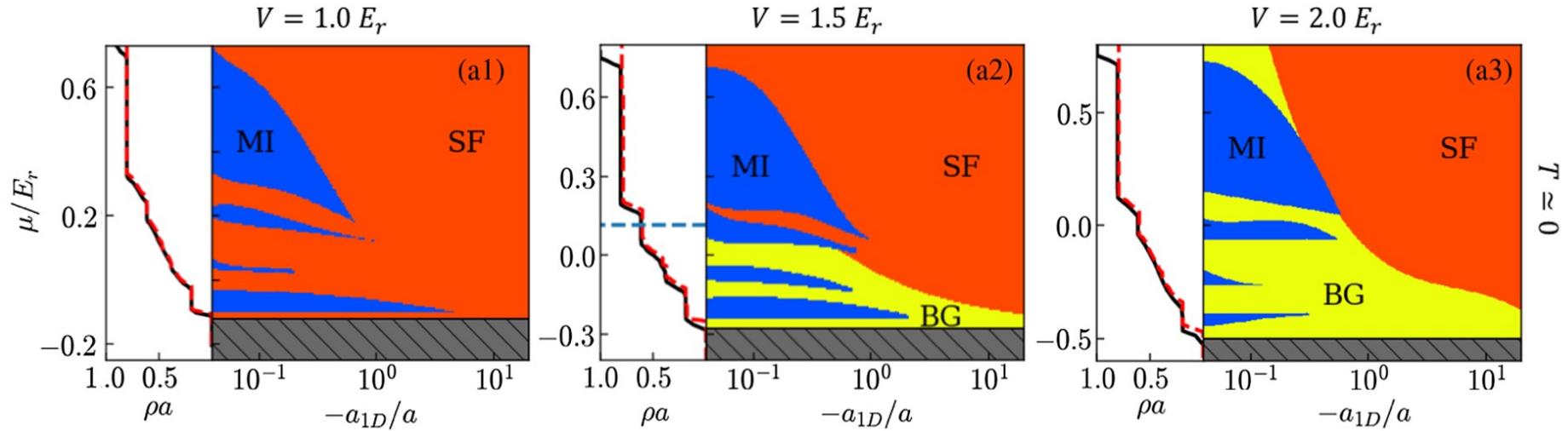


## What about finite temperatures?

A new regime : the normal fluid
It is a compressible insulator (just as the Bose glass)

→ Bose glass as long as no sizable finite-temperature
   effects are visible
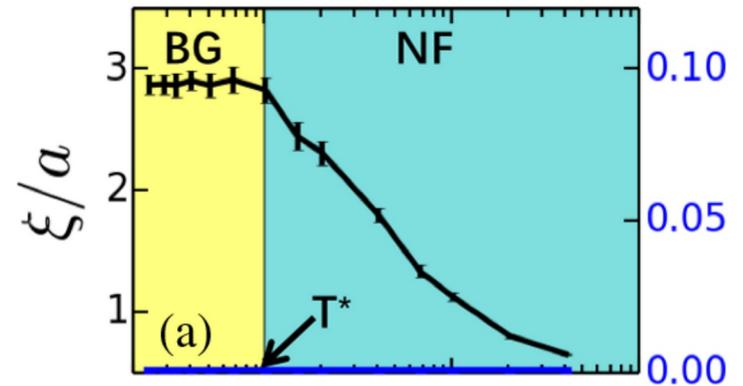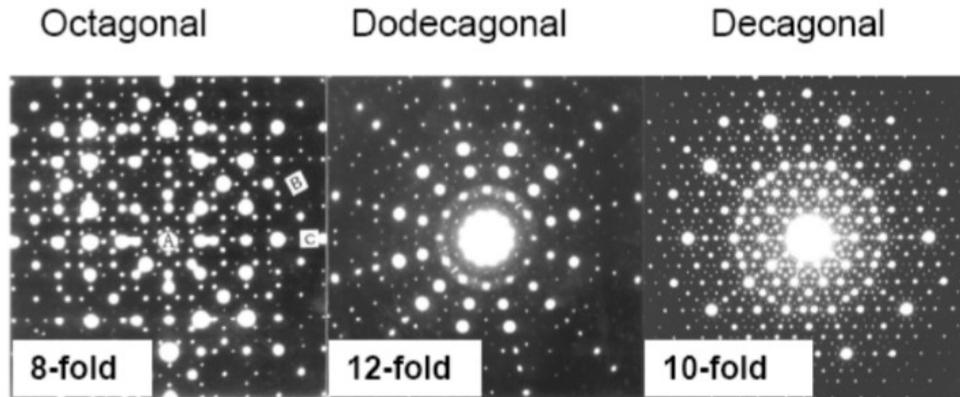
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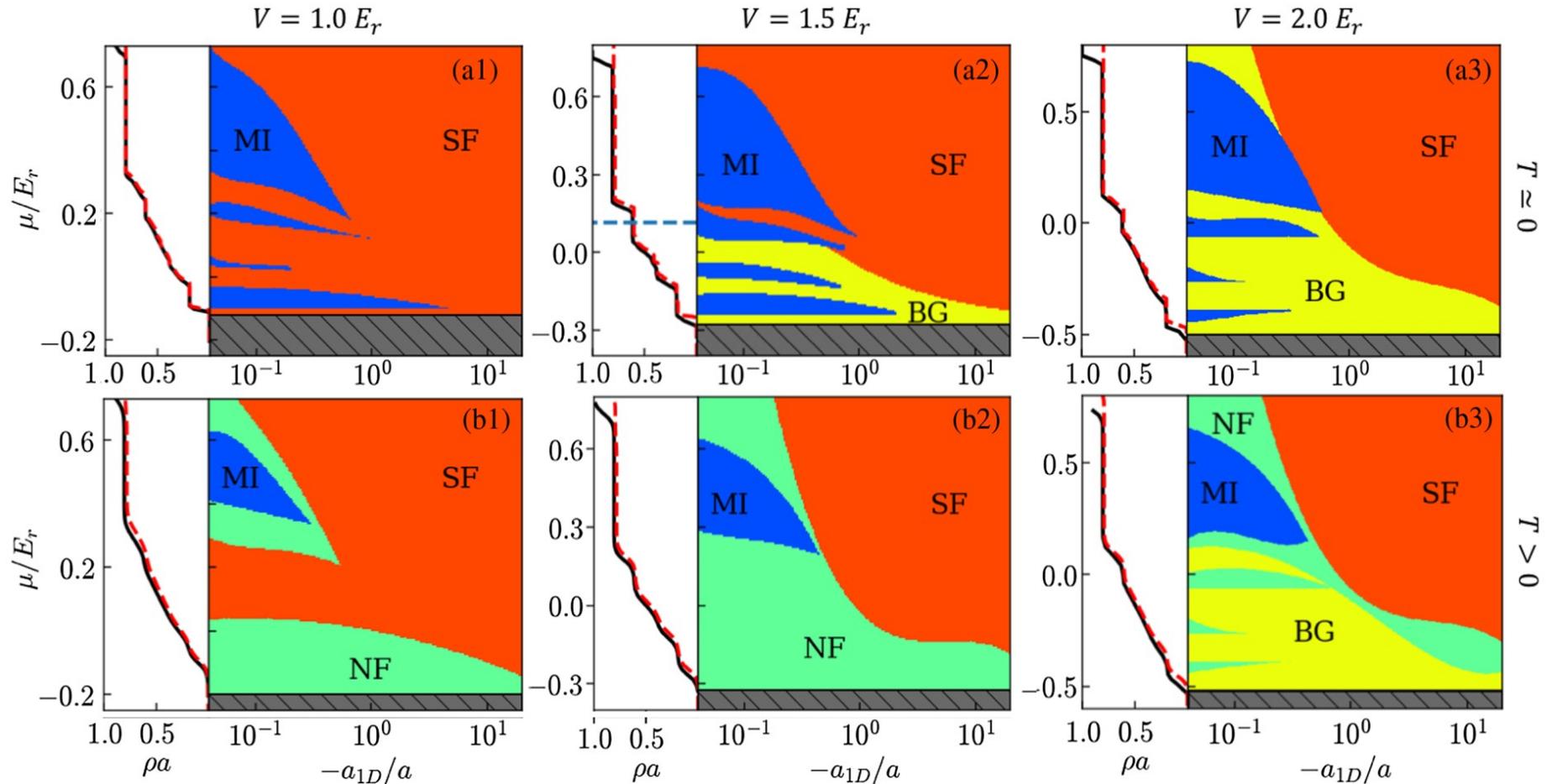
# Optical Quasicrystals : From 1D to 2D

## Quantum simulation of quasicrystals

Solids with long-range but nonperiodic order
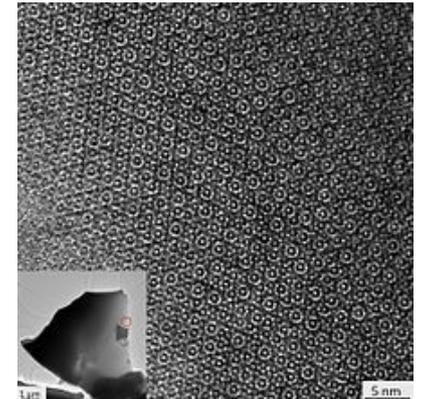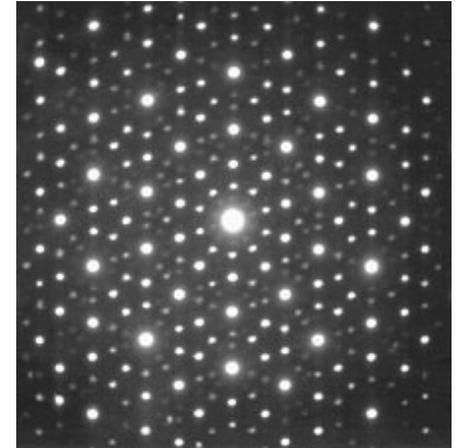
Discovered by chance using Bragg spectroscopy

Many intriguing physical properties



Lab synthesis of artificial quasicrystals

→ From correlated electrons to bosonic matter

Bragg spectroscopy :
long-range order
but nonperiodic



$Al_{71}Ni_{24}Fe_5$ quasicrystal
in a Khatyrka meteorite

# Optical Quasicrystal and Emergence of Quasiperiodic Order

Sanchez-Palencia & Santos, Phys. Rev. A **72**, 053607 (2005)
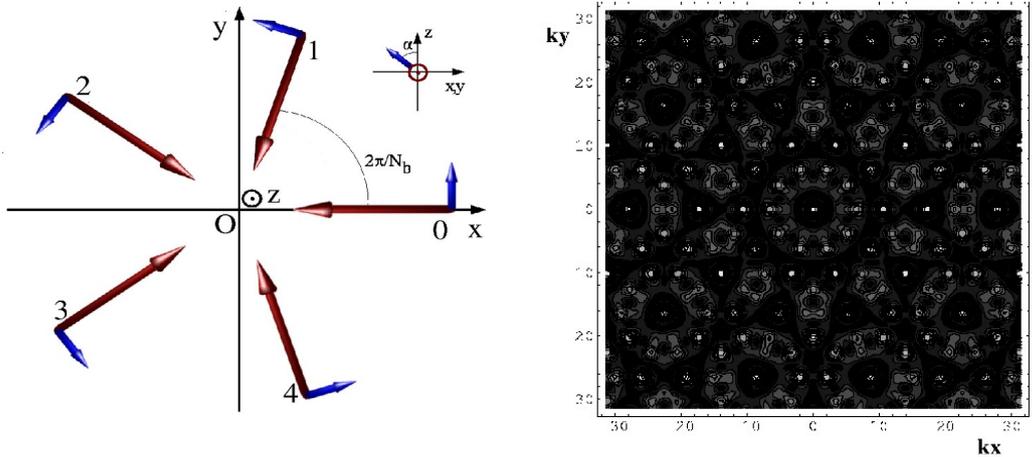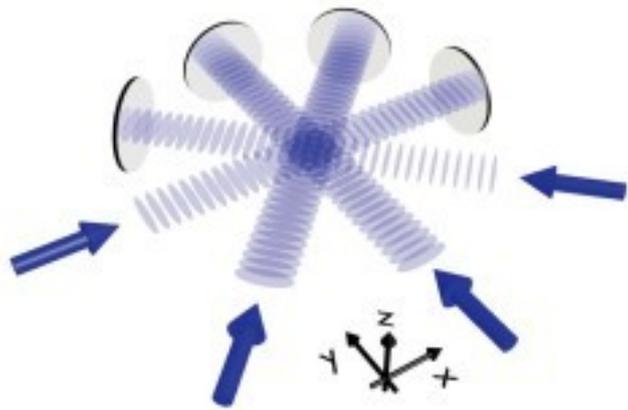


A potential with 10-fold rotational symmetry

$$V(\boldsymbol{r})=V_0\left|\sum_{1\leqslant j\leqslant 5}e^{i\,\boldsymbol{k}_j\cdot\boldsymbol{r}}\right|^2$$

The condensate contains all harmonics, owing to nonlinearity (interactions)

$$\mu\,\psi(\boldsymbol{r})=\frac{-\hbar^2\nabla^2}{2m}\,\psi(\boldsymbol{r})+V(\boldsymbol{r})\,\psi(\boldsymbol{r})+g\,|\psi(\boldsymbol{r})|^2\,\psi(\boldsymbol{r})$$

$$\psi(\boldsymbol{r})\propto\sqrt{1-\frac{V(\boldsymbol{r})}{\mu}}\sim1-\frac{V(\boldsymbol{r})}{2\mu}-\frac{V(\boldsymbol{r})^2}{8\mu^2}+\dots$$
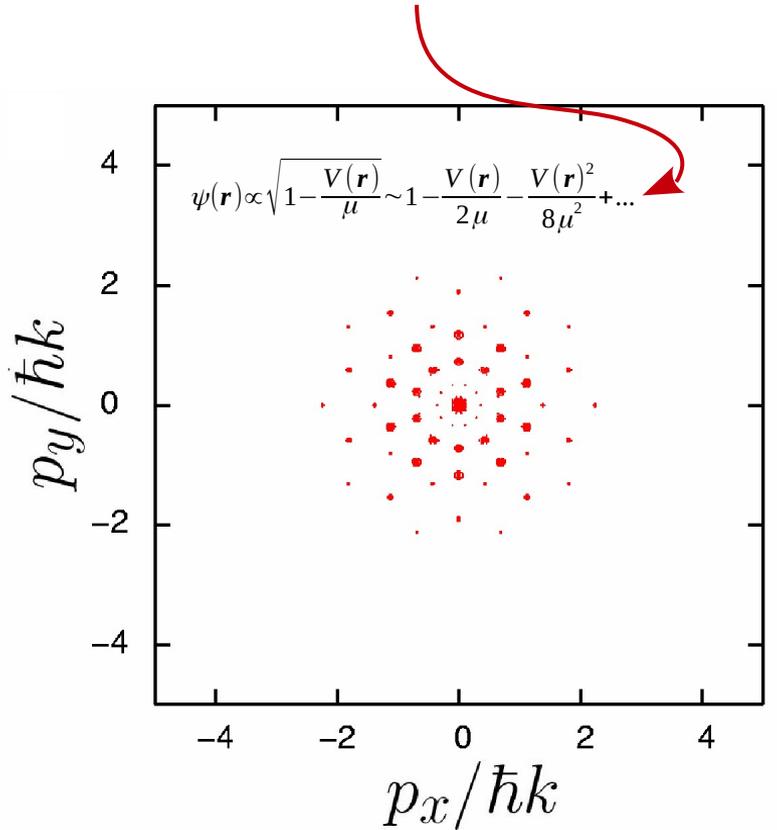
# Emerging Quasiperiodic Order from Ultracold Matterwaves
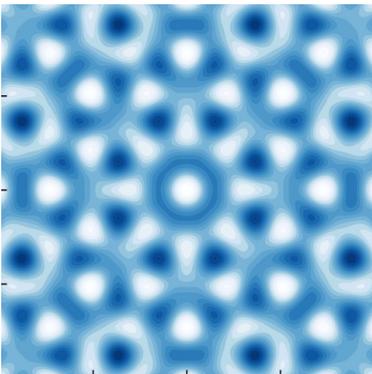


Jagannathan *et al.*, Europhys. Lett. **104**, 66003 (2013)

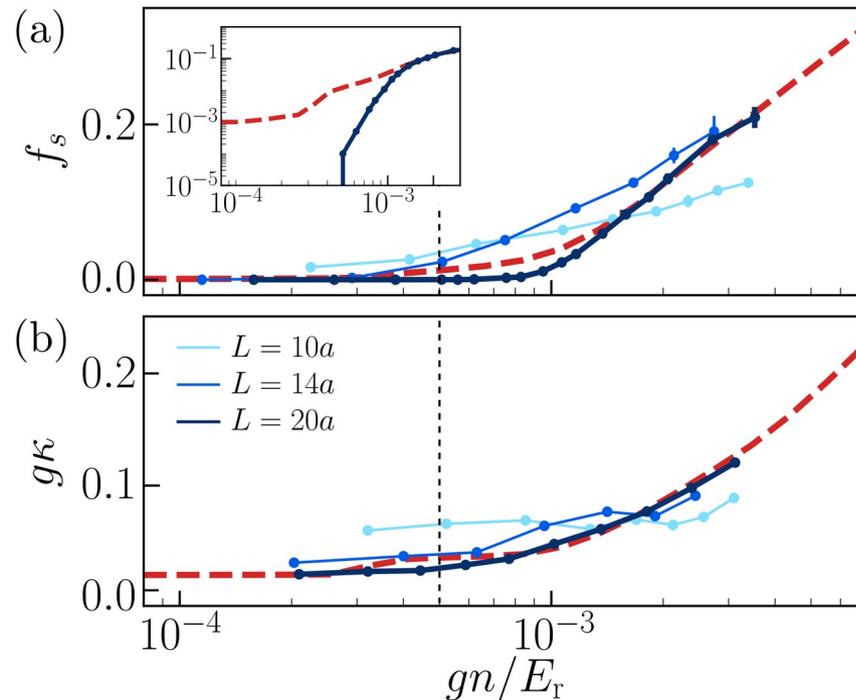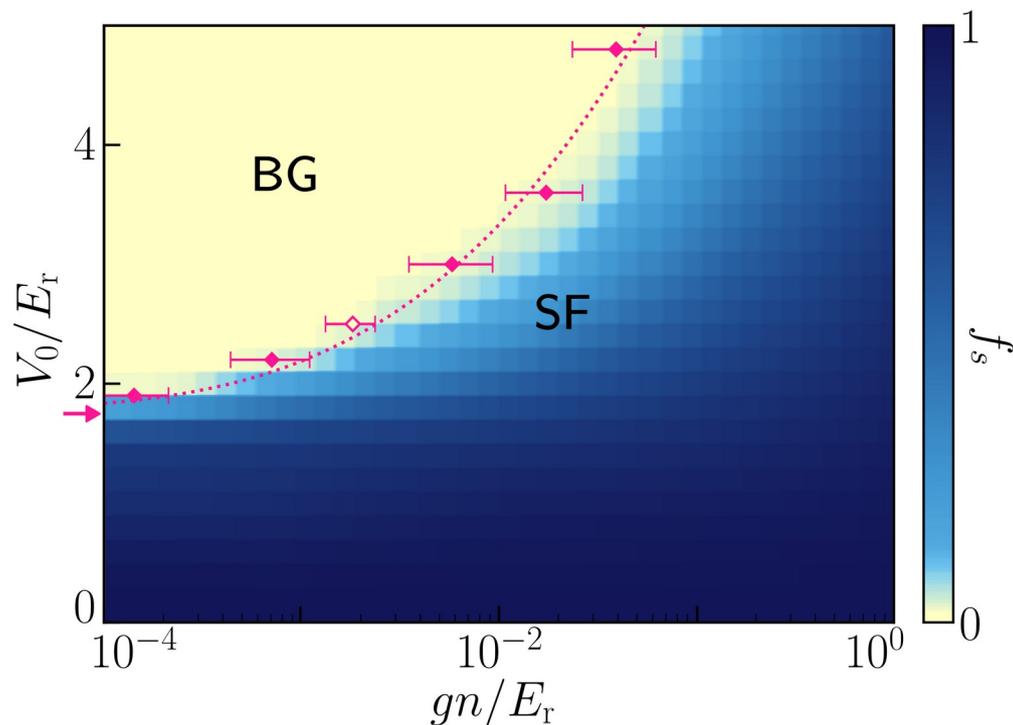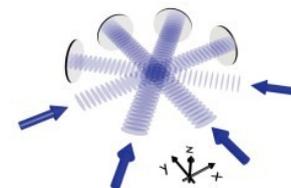Schneider's group, Phys. Rev. Lett. **122**, 110404 (2019)

Kapitza-Dirac diffraction

Quantum random walk in momentum space
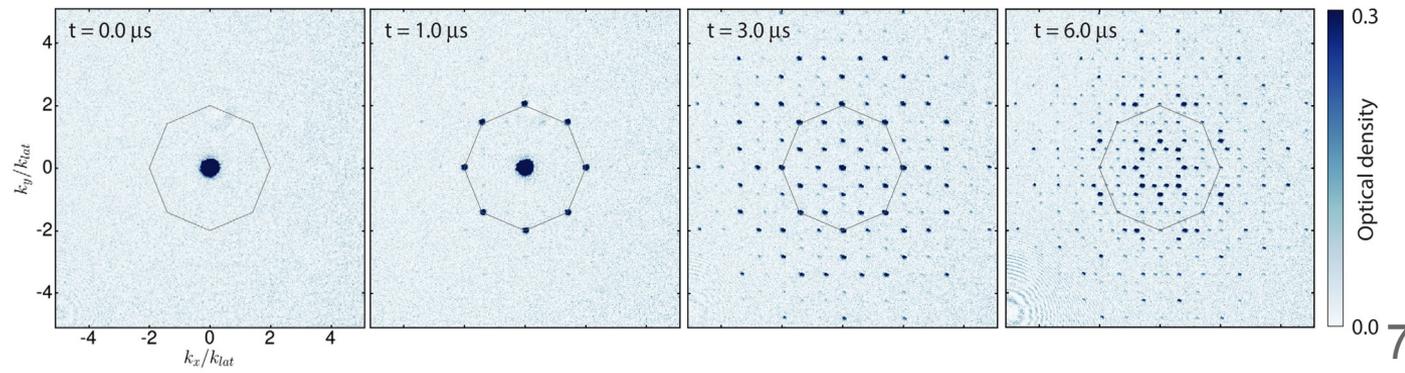


7

# Weakly-Interacting Bosons in a 2D Quasicrystal Lattice

Combining meanfield (Gross-Pitaevskii) and quantum Monte Carlo calculations

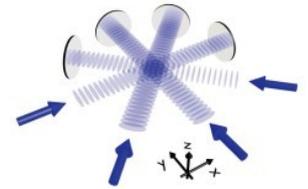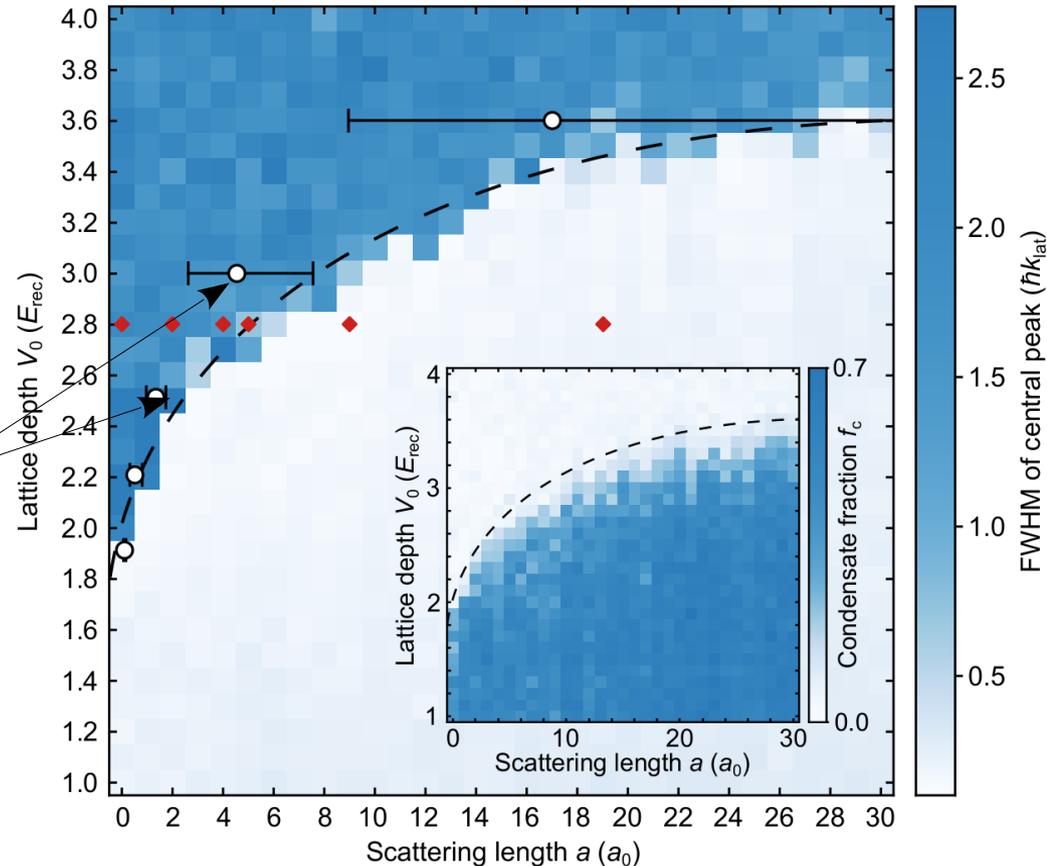# Weakly-Interacting Bosons in a 2D Quasicrystal Lattice
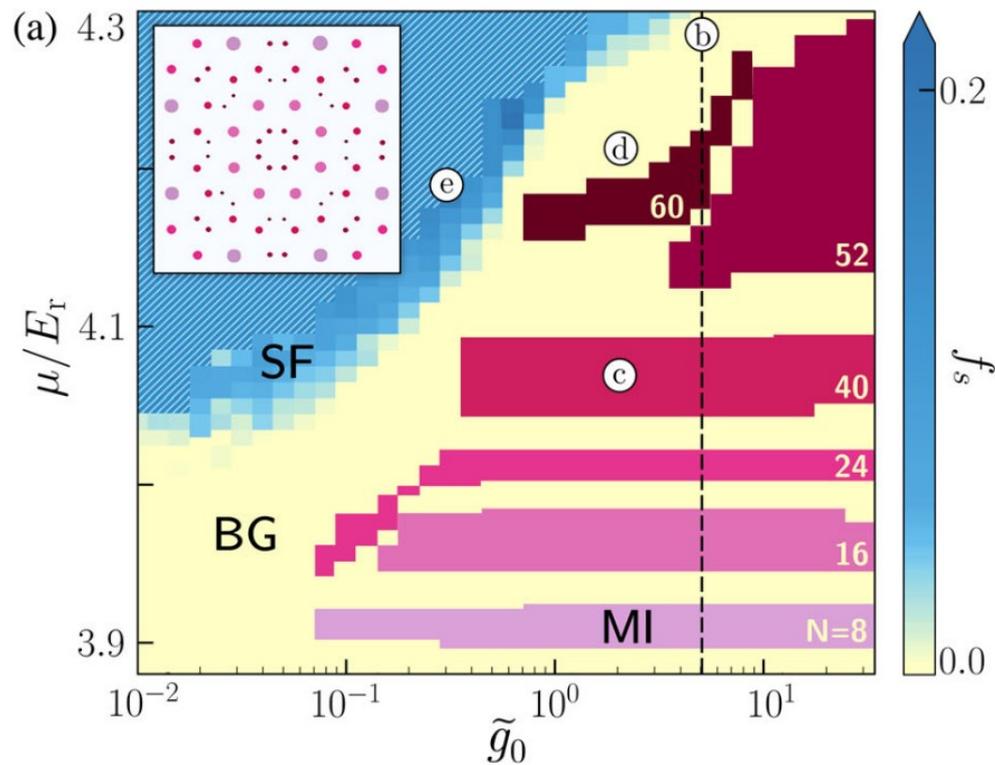
Coherence measurement

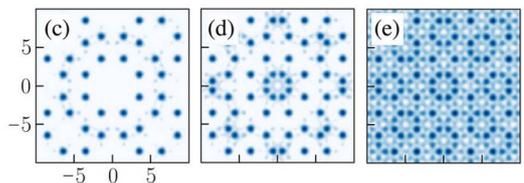Schneider's group
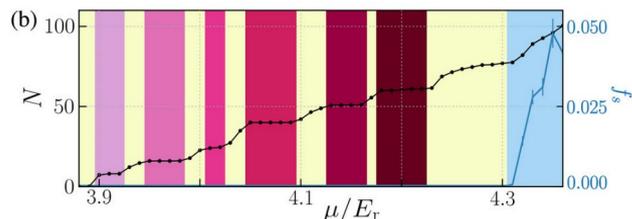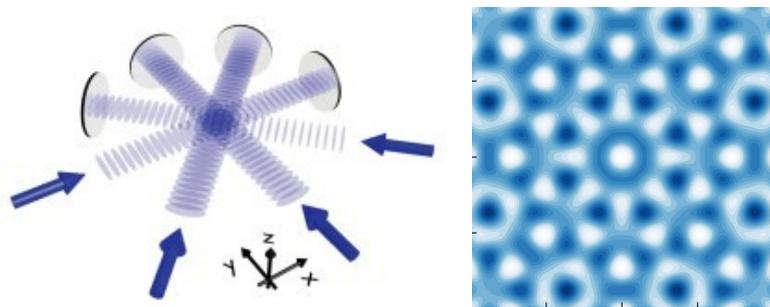[arXiv:2303.00737 (2023)]

Theoretical prediction

Gautier *et al.*,
Phys. Rev. Lett. **126**, 110401 (2021)

# Strongly-Interacting Bosons in a 2D Quasicrystal Lattice
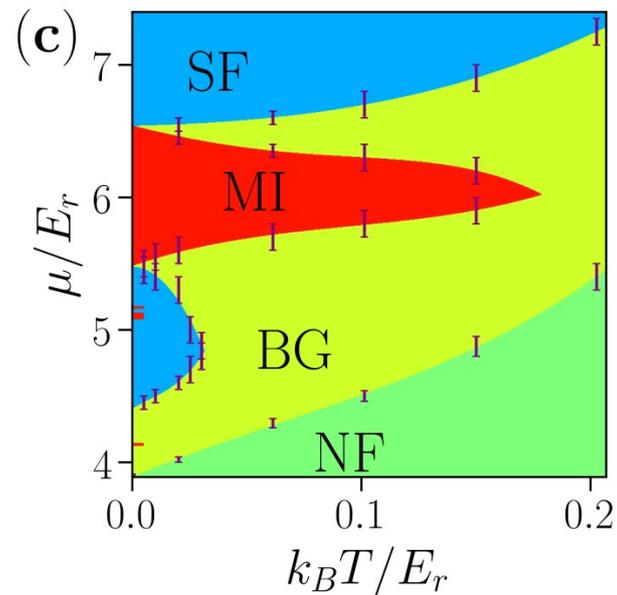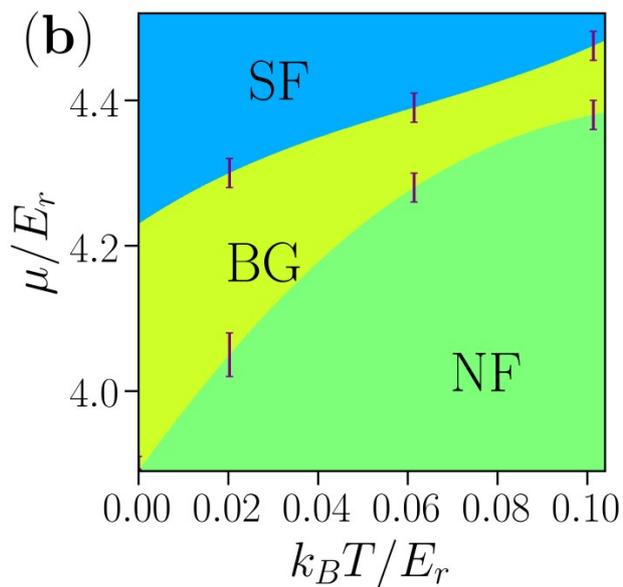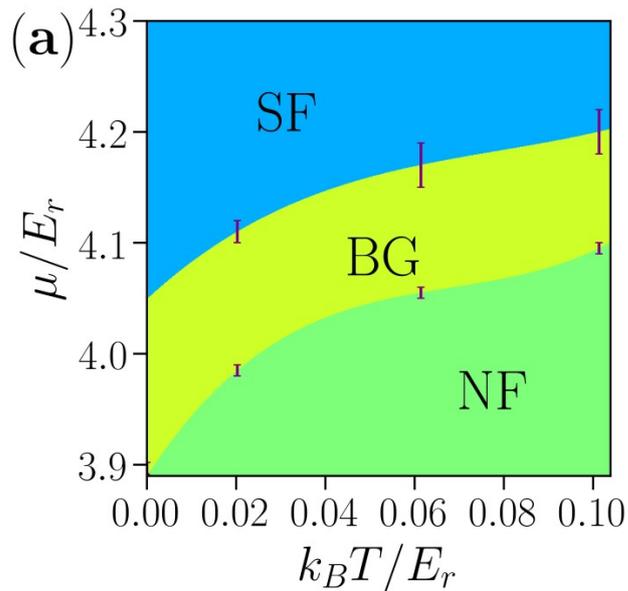
# Strongly-Interacting Bosons in a 2D Quasicrystal Lattice

Z. Zhu, H. Yao & L. Sanchez-Palencia, Phys. Rev. Lett. (2023)



Quantum phases survive to thermal fluctuations, up to significant temperatures

# Acknowledgements
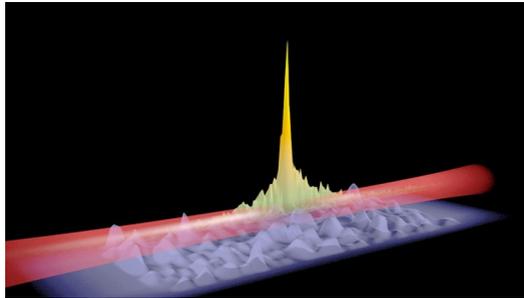
# Some of our (Recent) Activities

## Quantum disordered systems

### Anderson localization
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Billy *et al.*, Nature **453**, 891 (2008)
LSP and Lewenstein, Nat. Phys. **6**, 87 (2010)
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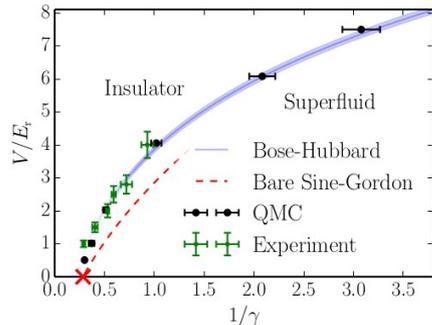
## Quantum physics in low dimensions
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Boéris *et al.*, Phys. Rev. A **93**, 011601(R) (2016)
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## Spreading of information in far-from-equilibrium correlated systems
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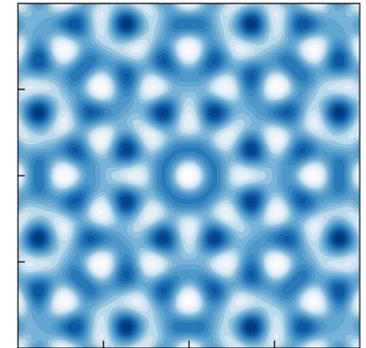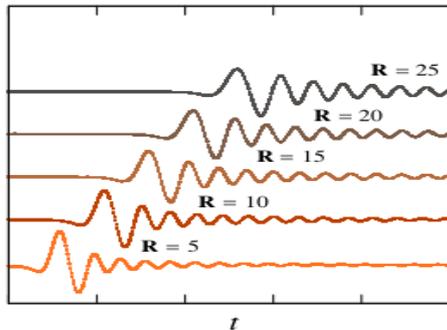
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Lellouch and LSP, Phys. Rev. A **90**, 061602(R) (2014)
Yao *et al.*, Phys. Rev. Lett. **123**, 070405 (2019)
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