High order harmonic generation: Quantum aspects of strong field physics

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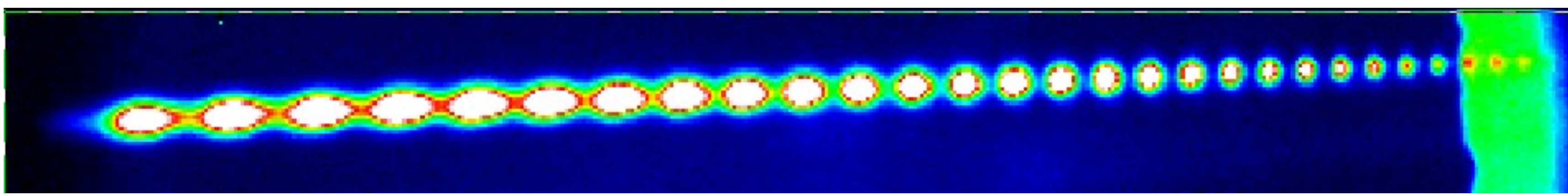
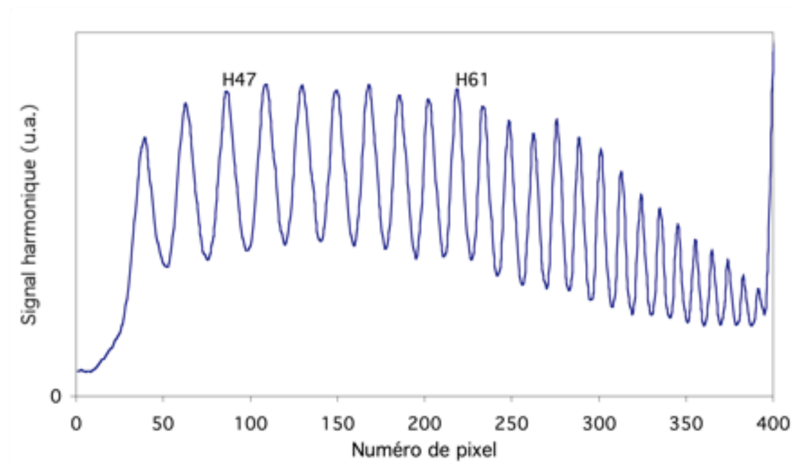


Typical harmonic spectrum in Neon



QUARMEN
Erasmus Mundus Joint Master
QUANTUM SCIENCE & TECHNOLOGY

Plan of the lecture:

- The most characteristic features of HHG, historical aspects

- How it looks like experimentally?

- The physical origin of the non linear polarization of atoms

- The problem of phase matching

- An ultrashort story of the attosecond structure

- Modern trends in quantum optics

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The most characteristic features of HHG, historical aspects:

- In 1985: the famous article by Strickland and Mourou opens the way to high power lasers with ultrashort pulse duration
- End 80's: studies about photo-ionization of atoms by intense lasers, electron/ion spectrometers, ATI spectra, first harmonic spectra

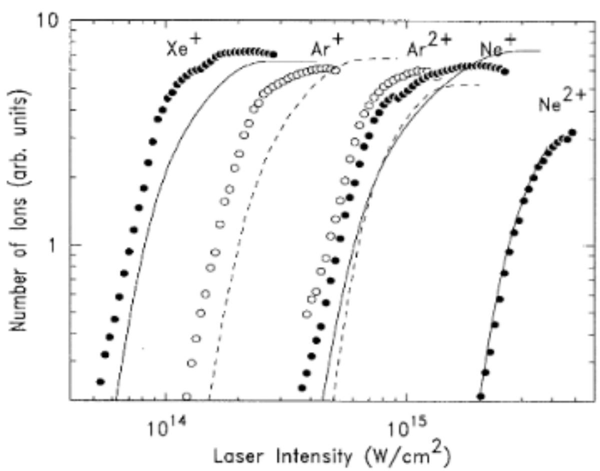


FIG. 3. Xe^+ (full circles), Ar^+, Ar^{2+} (open circles), Ne^+, and Ne^{2+} (full circles) ions as a function of the laser intensity. The lines (dashed for the Ar ions and solid for the Xe and Ne ions) are the predictions from tunnel ionization.

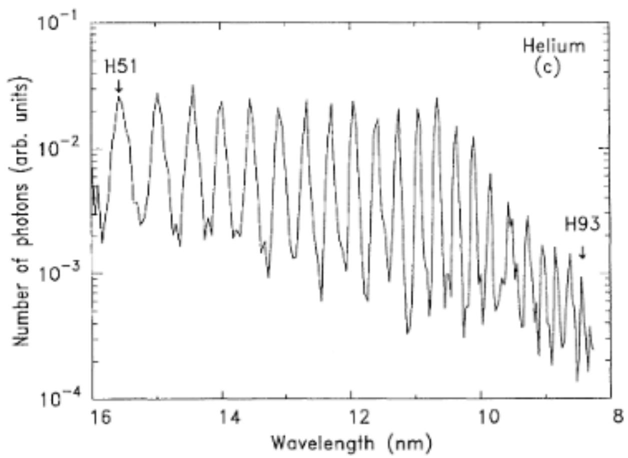


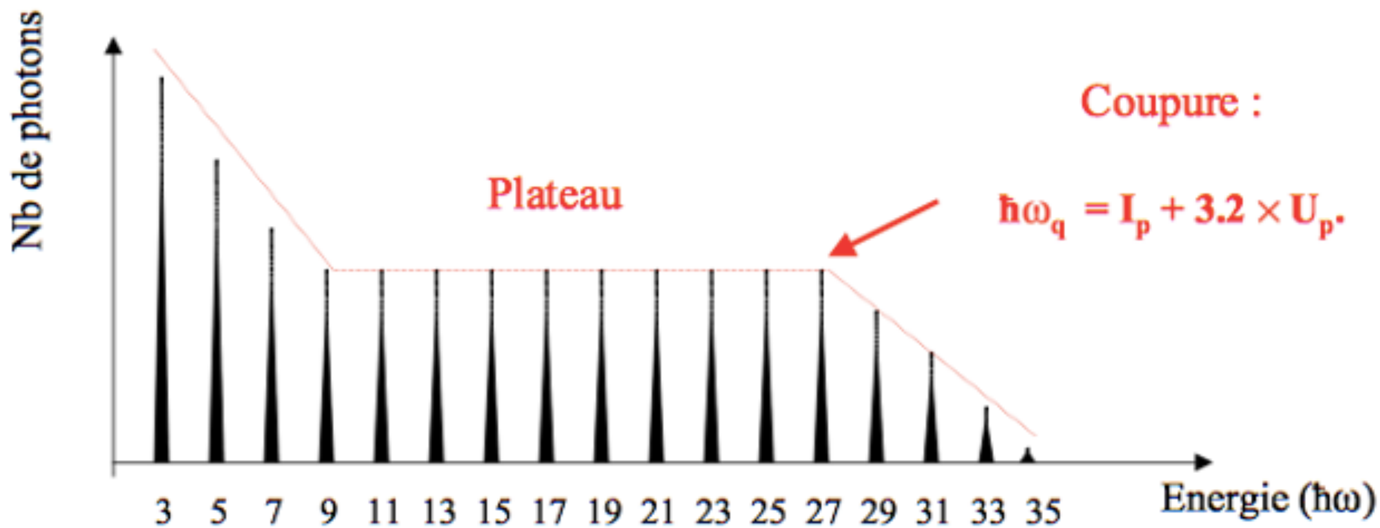
FIG. 4. Harmonic spectra in (a) Xe, (b) Ar, and (c) He. The laser intensity is approximately 10^{15} W/cm^2.

From C.G. Wahlström et al, PRA (1993)

The most characteristic features of HHG, historical aspects:

- 1993: the three step model is proposed to explain the physical origin of HHG, the cut-off law, still qualitative, semi-classical
- 1994/1995: the Lewenstein model, still quite “simple” but allows a more quantitative and quantic approach, explains the main features
- 1995: First prediction of the possibility for attosecond structure
- End 1990’s: The macroscopic aspects of HHG is studied both theoretically and experimentally
- 1999: The definition of the absorption limit for HHG
- Beginning 2000: first attosecond characterizations with the RABBIT method, attoscience is born
- Afterwards: The HHG source is used as a tool for applications in atomic physics, molecular physics, solid state physics, etc.
- Now: It can be a compact commercial source at high rep rate and people use it as a turn key black box
- The quantum nature of the state of light is studied and expected to be able to produce “Schrödinger cat” states that could be used for quantum computing and communication.

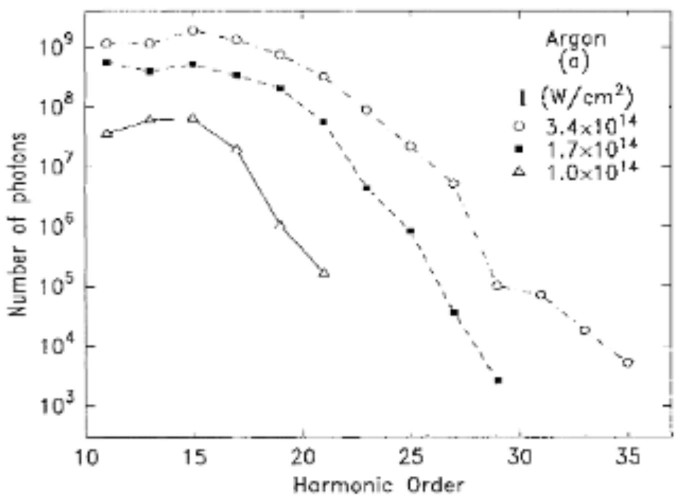
The most characteristic features of HHG:



$E_q = q\hbar\omega = \hbar\omega_q$

q is odd, there are 3 spectral regions

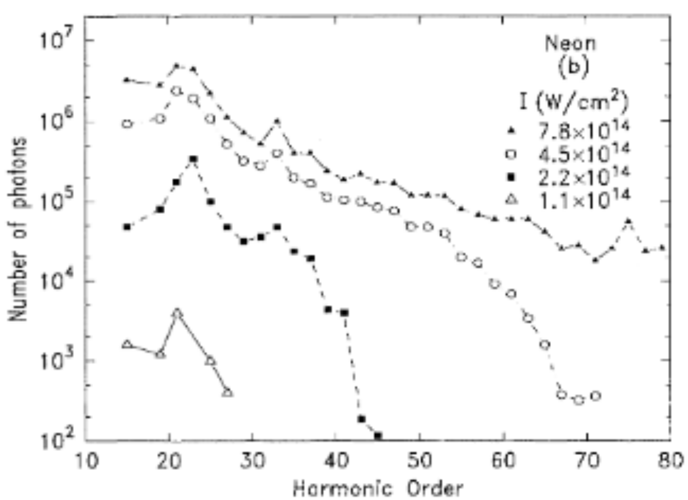
The most characteristic features of HHG: The cutoff law

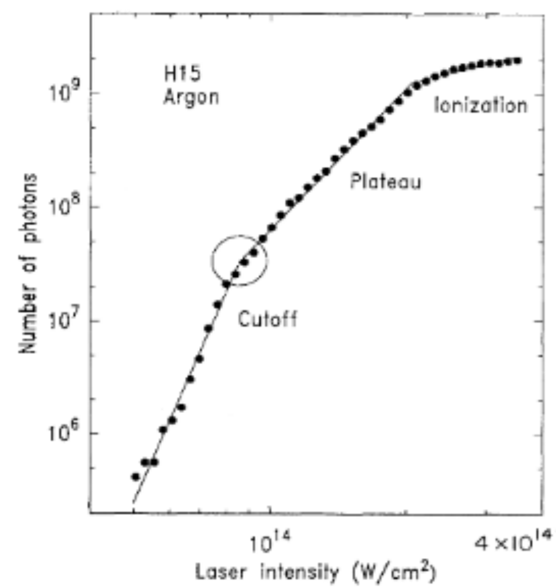


- $q_{max}\hbar\omega = I_p + 3{,}17\ U_p$

  $U_P = \frac{e^2}{8 m \varepsilon_0 c^3 \pi^2} \lambda^2 I$

Gaz considéré	Ip (eV)	I_{BSI} (W/cm^2)
Néon	21,56	8,20E+14
Argon	15,76	2,34E+14
Xénon	12,13	8,23E+13





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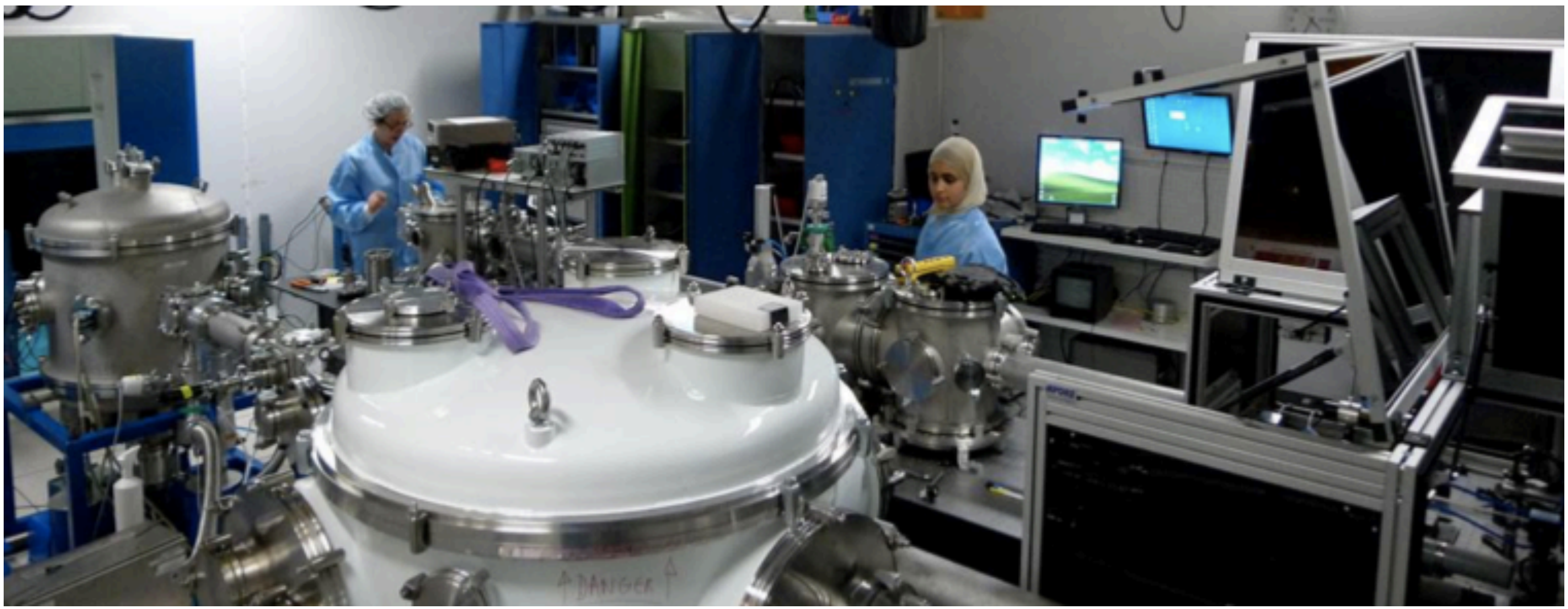
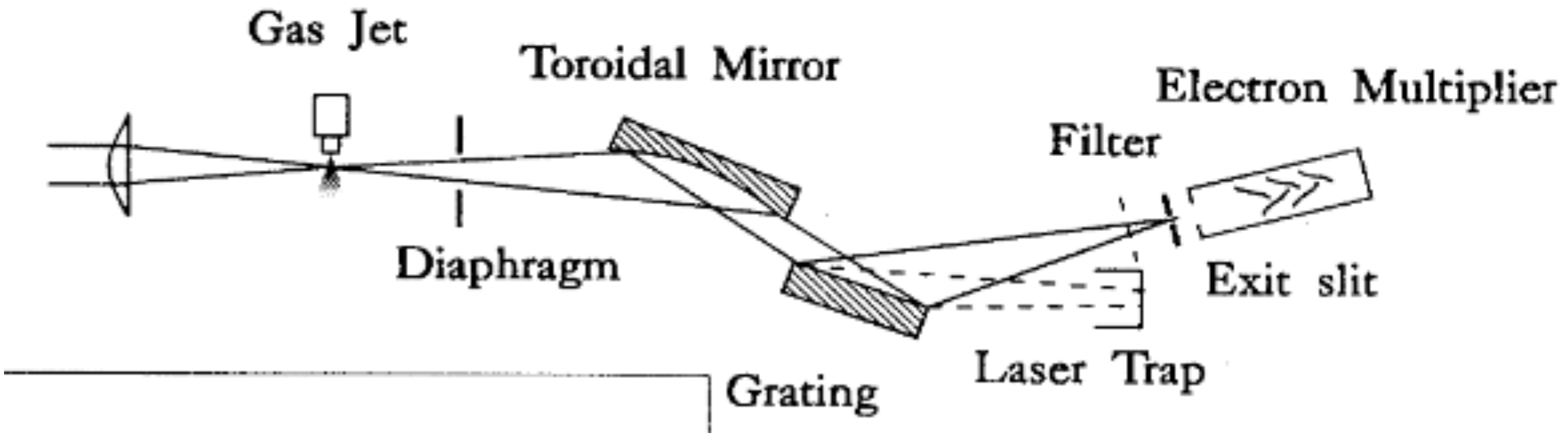
How it looks like experimentally?

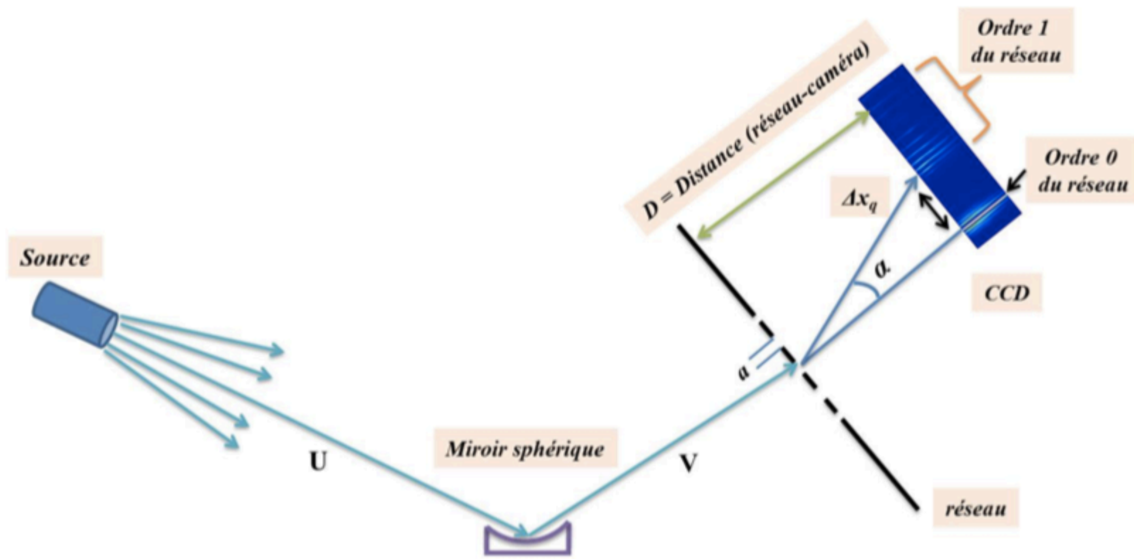


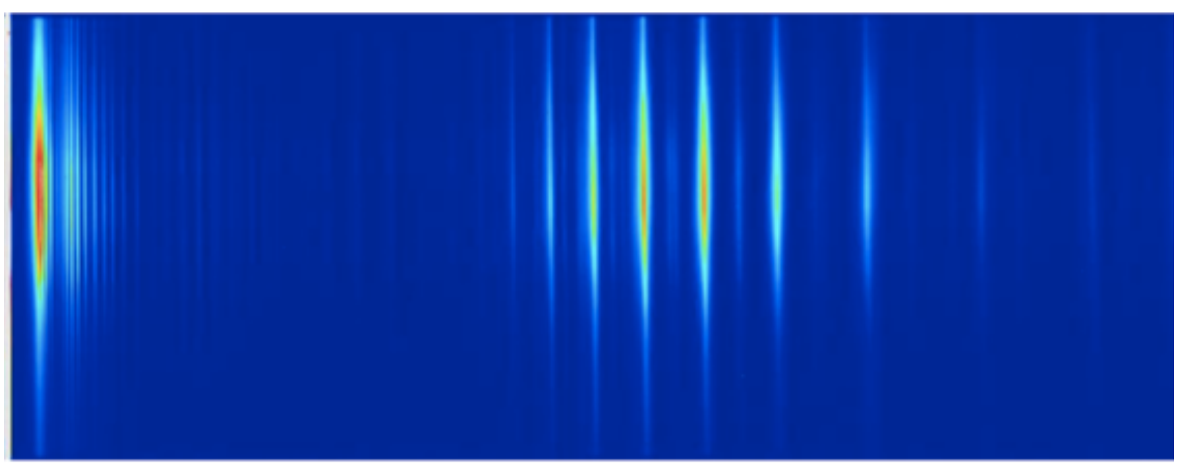
How it looks like experimentally?



- The gas is a rare gas, it can be a jet, a cell, pulsed or not
- The experiment is fully in vacuum
- The emission is on axis->filter is required
- The laser is focused and apertured (the size of the experiment depends on laser energy)
- Laser intensity is in the range 10^{14} W/cm^2, polarization is linear
- The best way to detect is a EUV spectrometer, microchannel plates, photodiode, CCD
- Efficiency is low: 10^{-4} is the maximum, goes down to 10^{-7} for short wavelengths
- Optical quality is good: low divergence, coherent beams, linear polarization

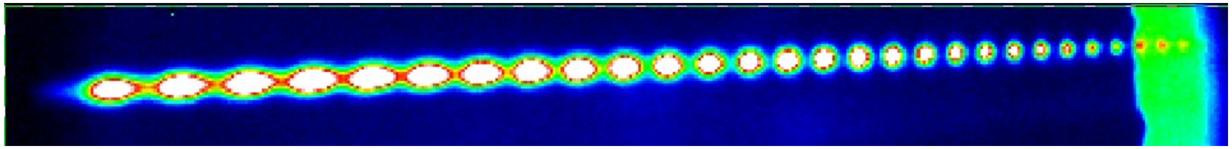
The most characteristic features of HHG: Typical experimental data



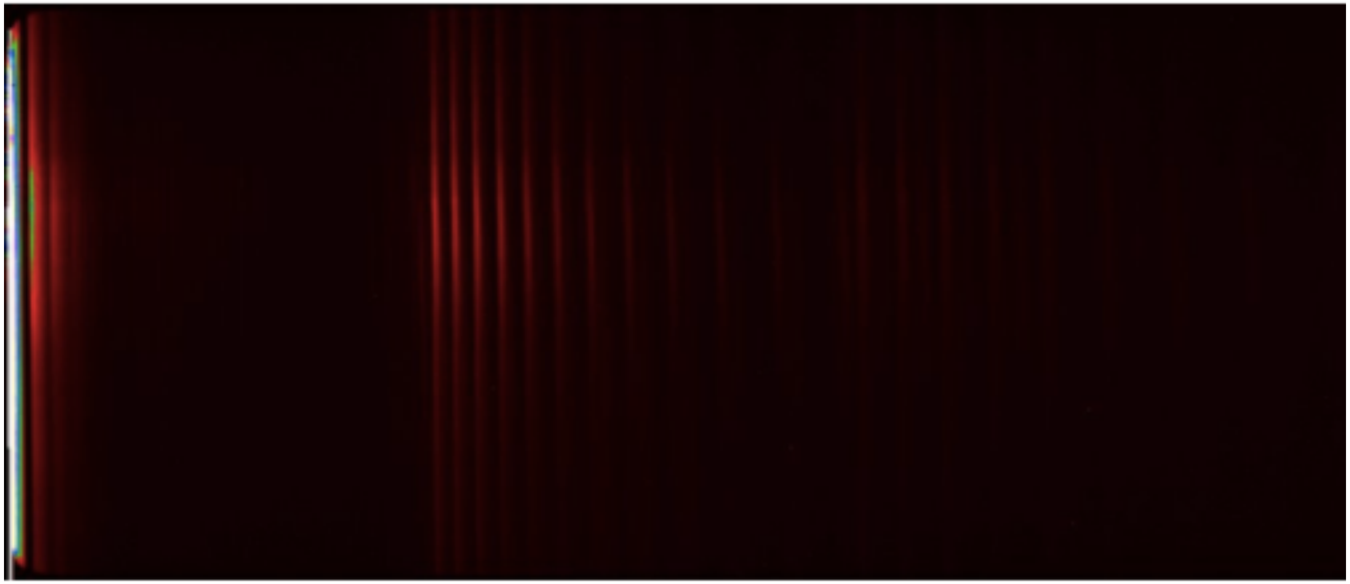


HHG spectrum in argon with a transmission spectrometer

The most characteristic features of HHG: Typical experimental data

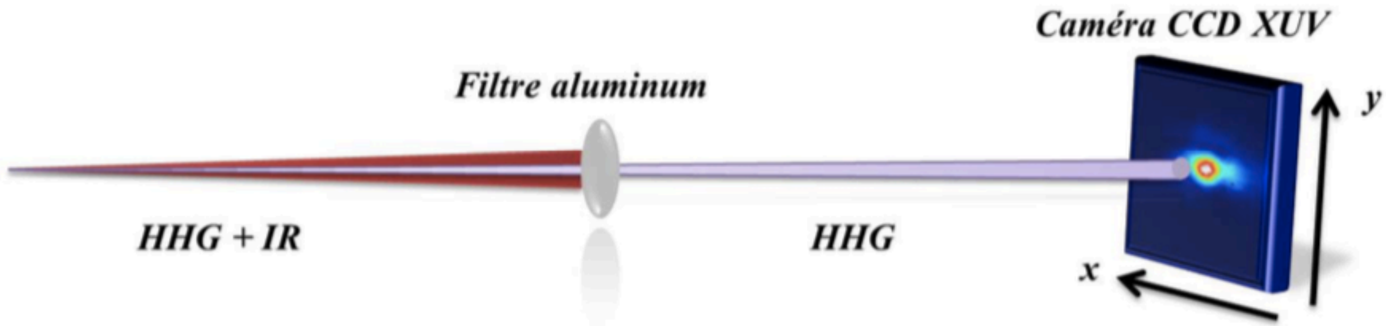


HHG spectrum in neon with an imaging spectrometer

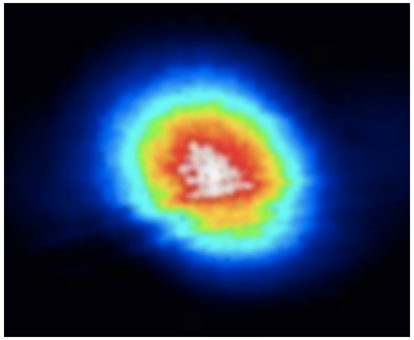


HHG spectrum in neon with a transmission spectrometer

The most characteristic features of HHG: Typical experimental data



HHG footprint after
Few meter
propagation



Gaz	Efficacité de conversion	q_{max}
Krypton	2-5 10^{-5}	21
Xénon	2-5 10^{-5}	21
Argon	10^{-5}	35
Néon	10^{-7}	81
Hélium	10^{-8}	301

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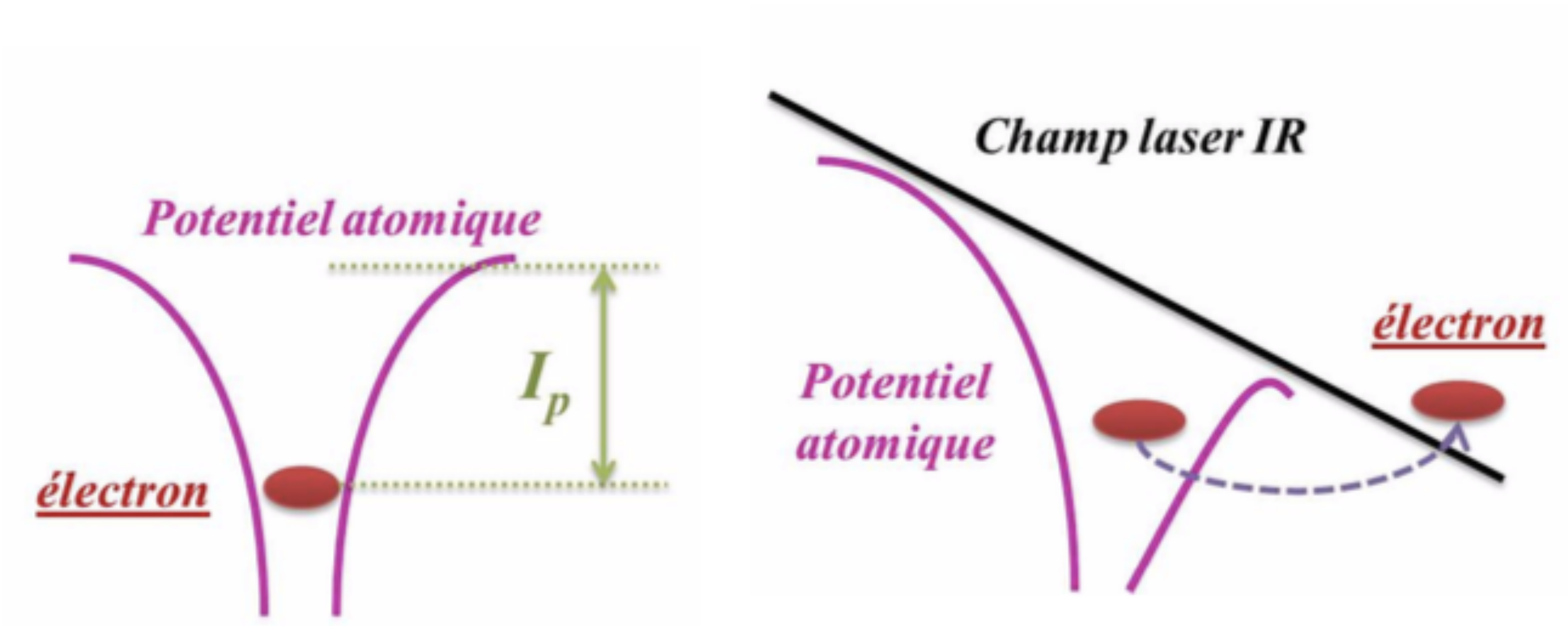
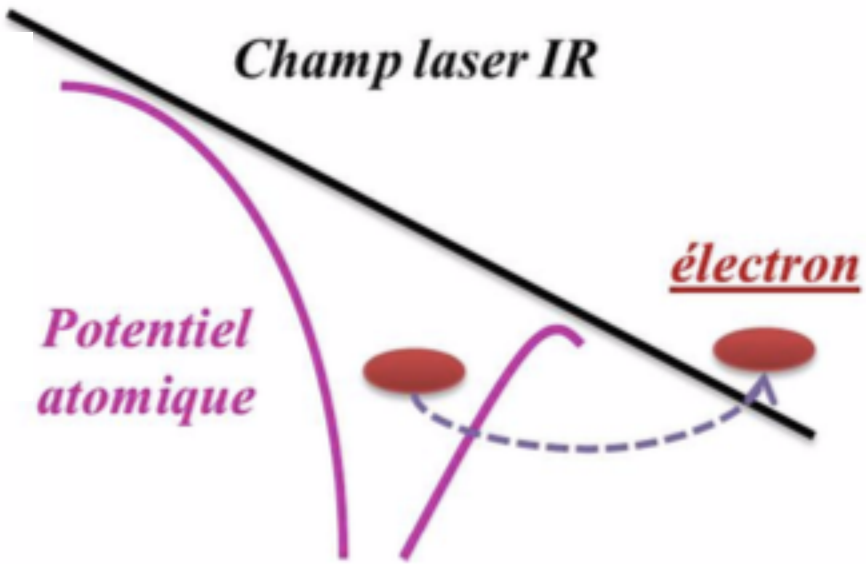
The physical origin of the non linear polarization Of atoms in strong laser fields



- In the bond state the electron energy is $-I_p$
- The atomic potential is $-Ze^2/4\pi\varepsilon_0 r$
- I_p is large as compared to 1 single laser photon energy (it should be multiphoton)
- U_p is large as compared to I_p
- The laser potential is -Ercosωt, r being the coordinate in the direction of propagation
- It must stay below the Intensity for barrier suppression (I_{BSI})

The physical origin of the non linear polarization Of atoms in strong laser fields

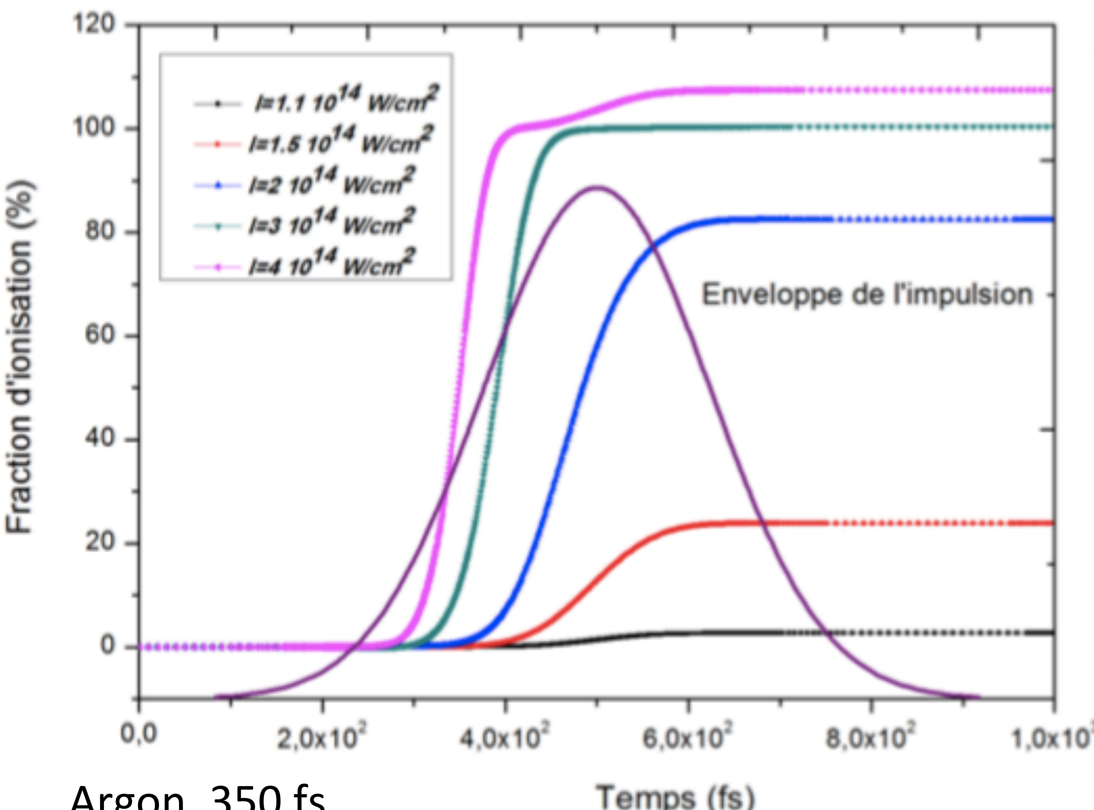
Gaz	I_P (eV)	I_{BSI} (W/cm^2)
Néon	21.56	8.20 10^{14}
Argon	15.75	2.34 10^{14}
Xénon	12.13	8.23 10^{13}



Up and IBSI can be calculated analytically

The physical origin of the non linear polarization Of atoms in strong laser fields

The ADK rates: it is a probability of tunnel barrier transmission per unit of time, analytical formula for the ionization rate as a function Of E_{laser} (the envelope) for a specific gas species



Argon, 350 fs

$$w_{ADK} = \sqrt{\frac{3n^{*}F}{\pi Z^{3}}}\,\frac{FD^{2}}{8\pi Z}\exp\left(-\frac{2Z^{3}}{3n^{*}F}\right)$$

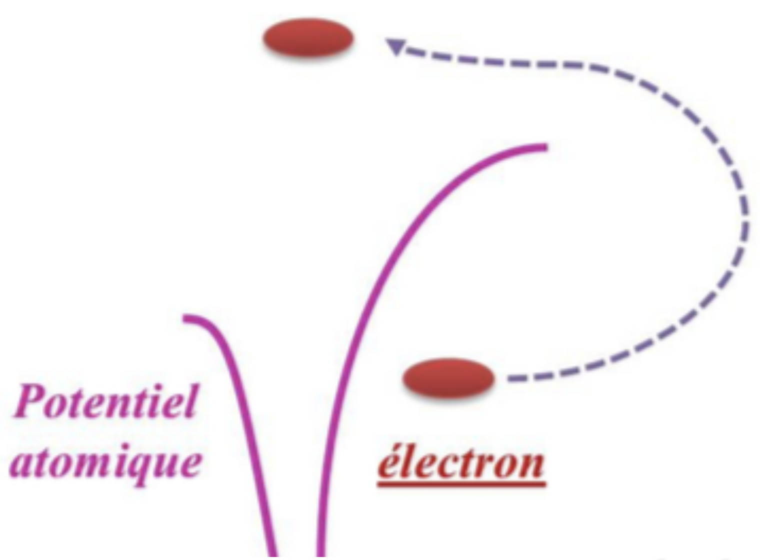
avec

$$n^{*} = \frac{Z}{\sqrt{2I_p}}$$

$$D = \left(\frac{4eZ^{3}}{Fn^{*4}}\right)^{n^{*}}$$

TDSE is closer to Reality:
Less approximations but more computation Time…

The physical origin of the non linear polarization Of atoms in strong laser fields



$$m_e a(t) = -eE_0 cos(\omega t)$$

$$E_c = \frac{e^2 E_0^2}{2m_e\omega^2}(sin(\phi) - sin(\phi_i))^2$$

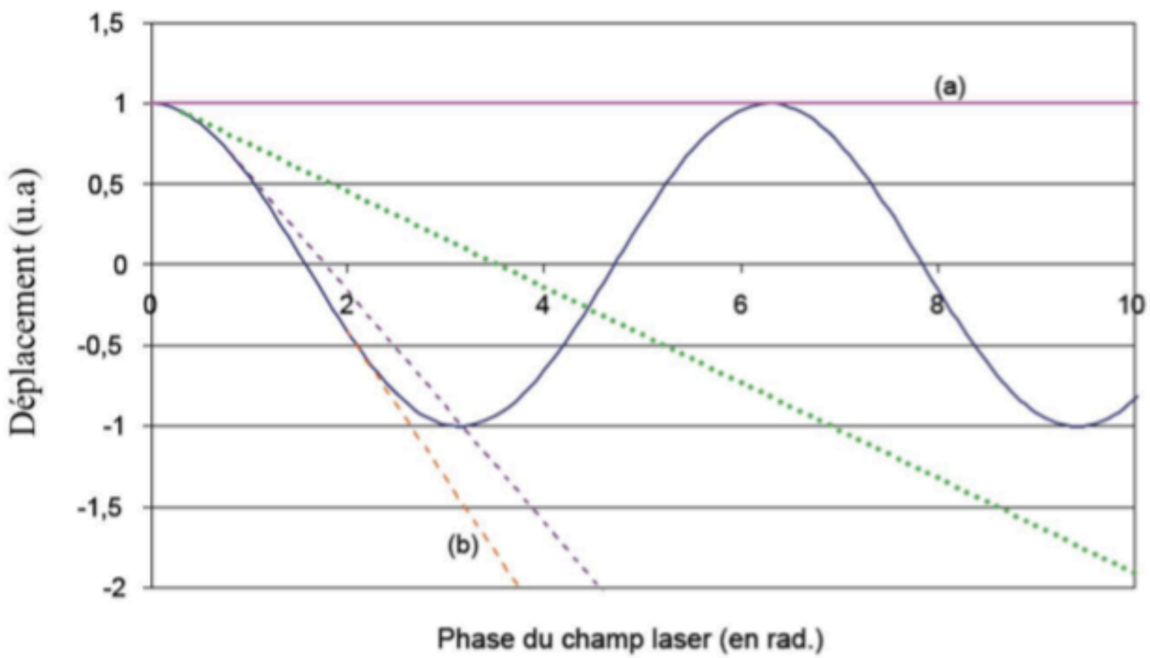
$$V(t, t_i) = -\frac{eE_0}{m_e\omega}(sin(\phi) - sin(\phi_i))$$

$$\frac{E_c}{U_P} = 2(sin(\phi) - sin(\phi_i))^2$$

$$x(t, t_i) = \frac{-eE_0}{m_e\omega^2}(cos(\phi_i) - cos(\phi) + sin(\phi_i)(\phi_i - \phi))$$

The physical origin of the non linear polarization Of atoms in strong laser fields

$$x(t,t_i) = \frac{-eE_0}{m_e\omega^2}\left(cos(\phi_i) - cos(\phi) + sin(\phi_i)(\phi_i - \phi)\right)$$

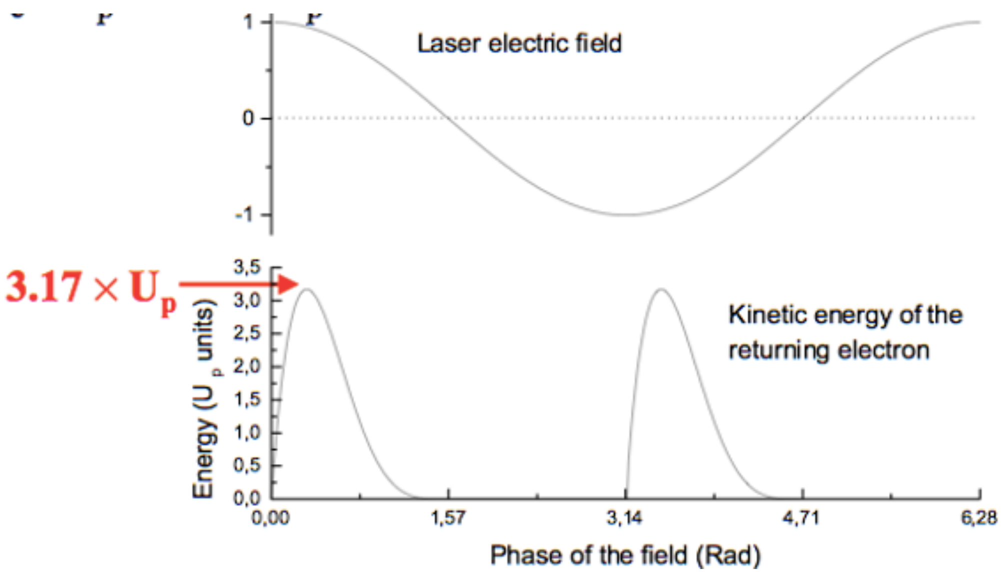


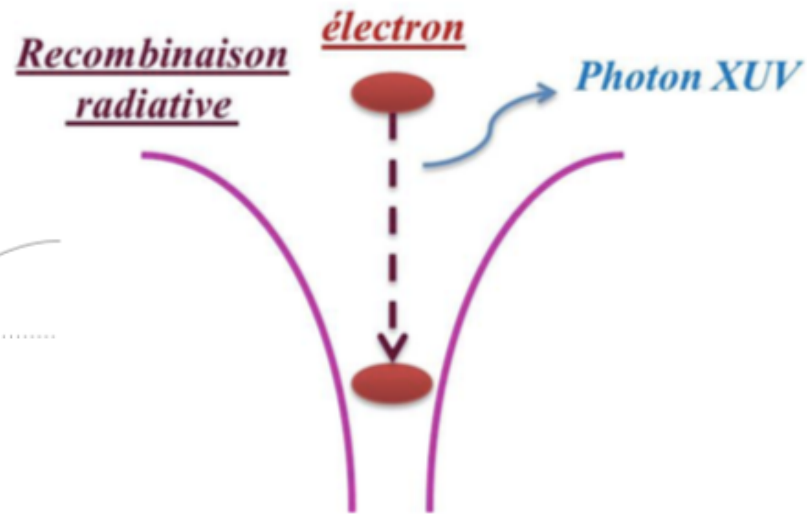
$$E_c = \frac{e^2E_0^2}{2m_e\omega^2}(sin(\phi) - sin(\phi_i))^2$$

$$\frac{E_c}{U_P} = 2(sin(\phi) - sin(\phi_i))^2$$

Two main trajectories for the same Kinetic energy gain :
The long one is emitted earlier and recombine later

The physical origin of the non linear polarization Of atoms in strong laser fields

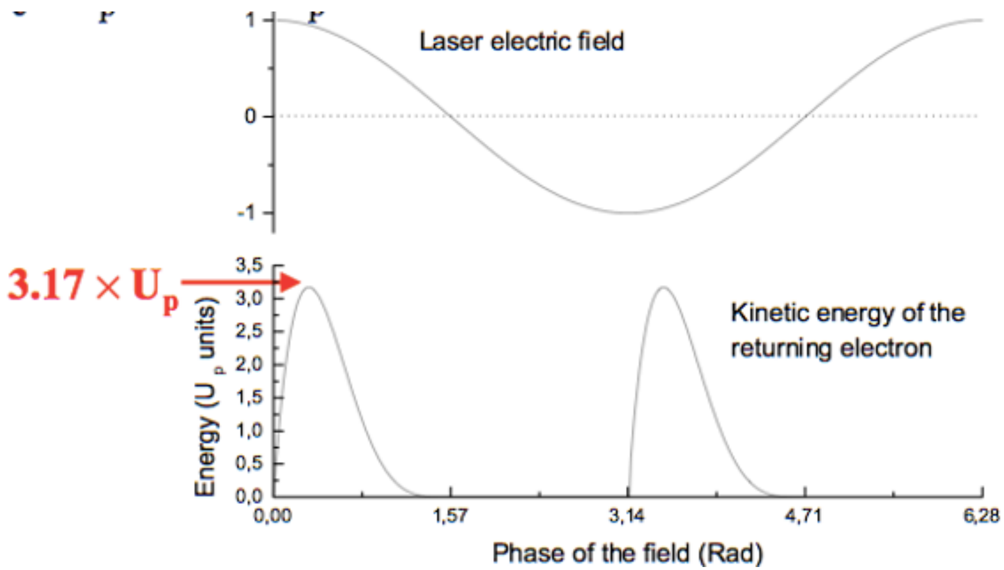




The three step model explains the cut-off law

The two trajectories are clearly visible, they converge in the cutoff region

Why is it odd harmonic?



- The harmonic dipole changes Sign every half period:
- This is due to the spherical symetry of the atom and the fact That the dipole is related to The algebric distance to the Nucleus along the polarization
- If one single harmonic burst: no Harmonic structure

$$\tilde{S}(\omega) = \left| \sum_{\tau} E(\omega) \exp i(\omega\tau + \varphi) \right|^2$$

$$\tilde{S}(\omega) = |E(\omega)|^2 |1 - \exp i\omega\tau|^2$$

$$\tau = \frac{\pi}{\omega_{IR}}; q = \frac{\omega_{XUV}}{\omega_{IR}}; \tilde{S}(\omega) = |E(\omega)|^2 (2 - 2\cos q\pi)$$

"Theory of high harmonic generation by low-frequency laser fields", Phys Rev A vol 49, num 3, page 2117, (1994).

$$i\hbar\frac{\partial \mid \Psi(t,x)\rangle}{\partial t} = (-\frac{1}{2}\nabla^2 + V(x) - Ecos(t).x(t)) \mid \Psi(t,x)\rangle$$

Kinetic energy

Atomic potential

Laser energy term

If we assume:

$$\mid \Psi(t,x)\rangle = e^{i\frac{I_p t}{\hbar}}(a(t) \mid 0\rangle + \int d^3v b(v,t) \mid v\rangle)$$

Inject, calculate and do the scalar product with the state v gives:

$$\frac{\partial b}{\partial t} = -i(\frac{v^2}{2} + I_p)b(v,t) - Ecost\frac{\partial b}{\partial v_x} + iEa(t)costd_x(v)$$

Scalar product with the state 0 gives:

$$\dot{a} = iEcost\int d^3v d_x(v)b(v,t)$$

$$d_x(v) = \langle v \mid x \mid 0\rangle$$

"Theory of high harmonic generation by low-frequency laser fields", Phys Rev A vol 49, num 3, page 2117, (1994).

$$b(\vec{v},t) = i\int_0^t dt' E cos t' d_x(\vec{v}+\vec{A}(t)-\vec{A}(t'))exp(-i\int_{t'}^t dt''[I_p + \frac{(\vec{v}+\vec{A}(t)-\vec{A}(t''))^2}{2}])$$

We are looking for: $x(t) = \langle \Psi(t) \mid x \mid \Psi(t) \rangle$.

$$(a^*(t)\langle 0 \mid + \int d^3v b^*(v,t)\langle v \mid) x (a(t) \mid 0\rangle + \int d^3 v b(v,t) \mid v\rangle)$$

After some maths, t' is ionization time, t is recombination time, v=p-A:

$$x(t) = i\int_0^t dt' \int d^3p E cos t' d_x(p-A(t'))d_x^*(p-A(t))exp(-iS(p,t,t'))$$

$$S(p,t,t') = \int_{t'}^t dt''(I_p + \frac{(P-A(t''))^2}{2})$$

"Theory of high harmonic generation by low-frequency laser fields", Phys Rev A vol 49, num 3, page 2117, (1994).

$$x(t) = i\int_0^\infty d\tau(\frac{\pi}{\epsilon + i\frac{\tau}{2}})^{\frac{3}{2}} d_x^*(p_{st}(t,\tau)-A(t))d_x(p_{st}(t,\tau)-A(t-\tau))Ecos(t-\tau)exp(-iS_{st}(t,\tau))+c.c.$$

Integration is done over all times spent in the continuum
With:

$$p_{st}(t,\tau) = \frac{E}{\tau}[cos(t) - cos(t-\tau)]$$

$$S_{st}(t,\tau) = \int_{t-\tau}^{t} dt" \frac{(p_{st} - A(t"))^2}{2} + I_p.$$

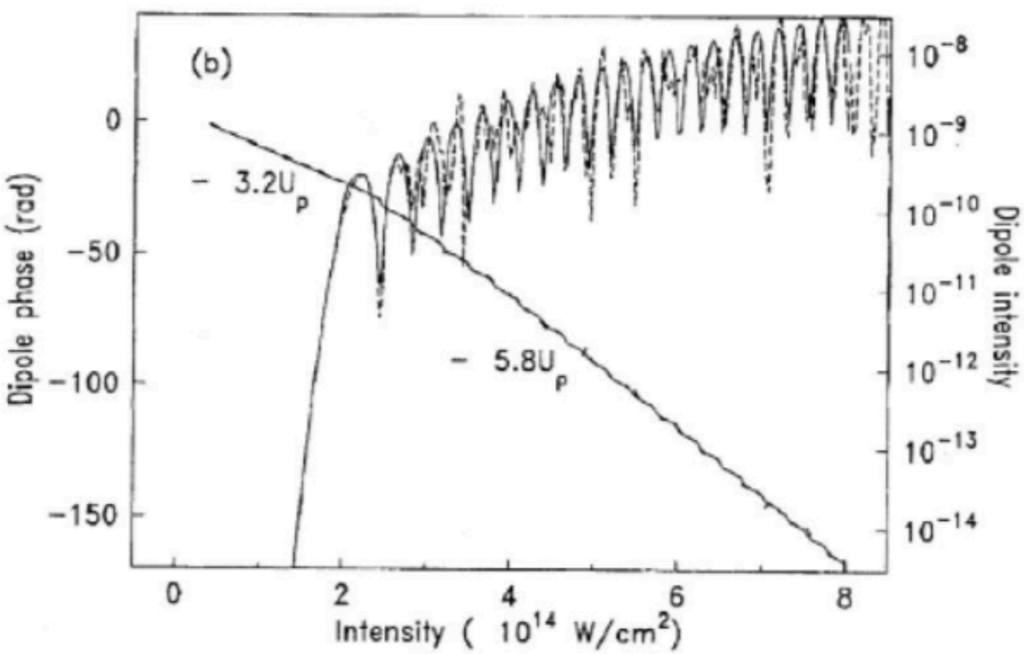
$$S_{st}(t,\tau) = (I_p + U_p)\tau - \frac{2U_p}{\tau}(1 - cos\tau) - U_p C(\tau) cos(2t - \tau)$$

$$C(\tau) = sin(\tau) - 4\frac{sin^2(\frac{\tau}{2})}{\tau}$$

"Theory of high harmonic generation by low-frequency laser fields", Phys Rev A vol 49, num 3, page 2117, (1994).

$$x(t) = i\int_0^\infty d\tau\left(\frac{\pi}{\epsilon + i\frac{\tau}{2}}\right)^{\frac{3}{2}} d_x^*(p_{st}(t,\tau) - A(t))\, d_x(p_{st}(t,\tau) - A(t-\tau))\, E\cos(t-\tau)\exp(-iS_{st}(t,\tau)) + c.c.$$

The Fourier transform of x(t) gives the harmonic spectrum in amplitude and phase



H45 in neon

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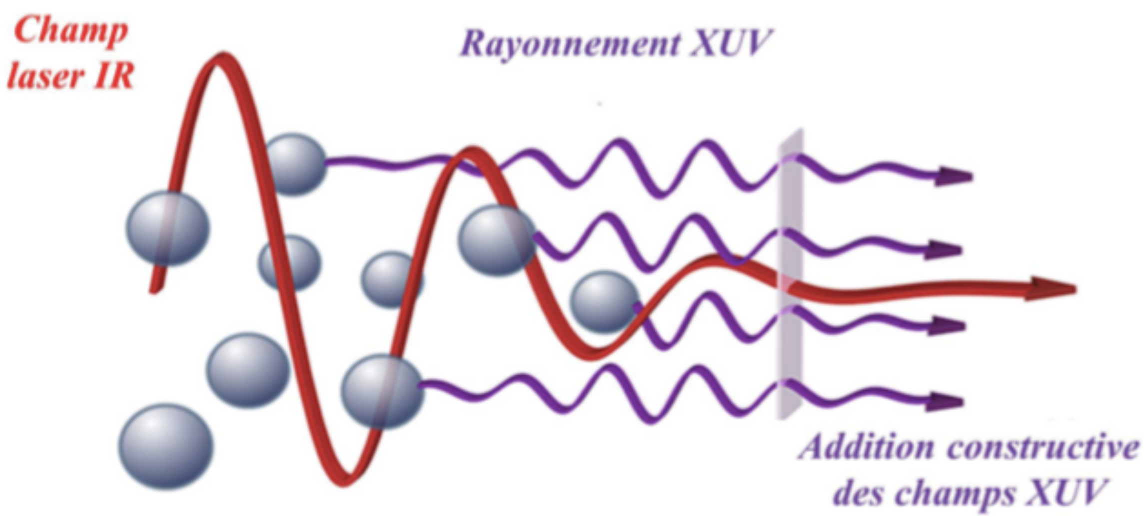
Macroscopic aspect of HHG:



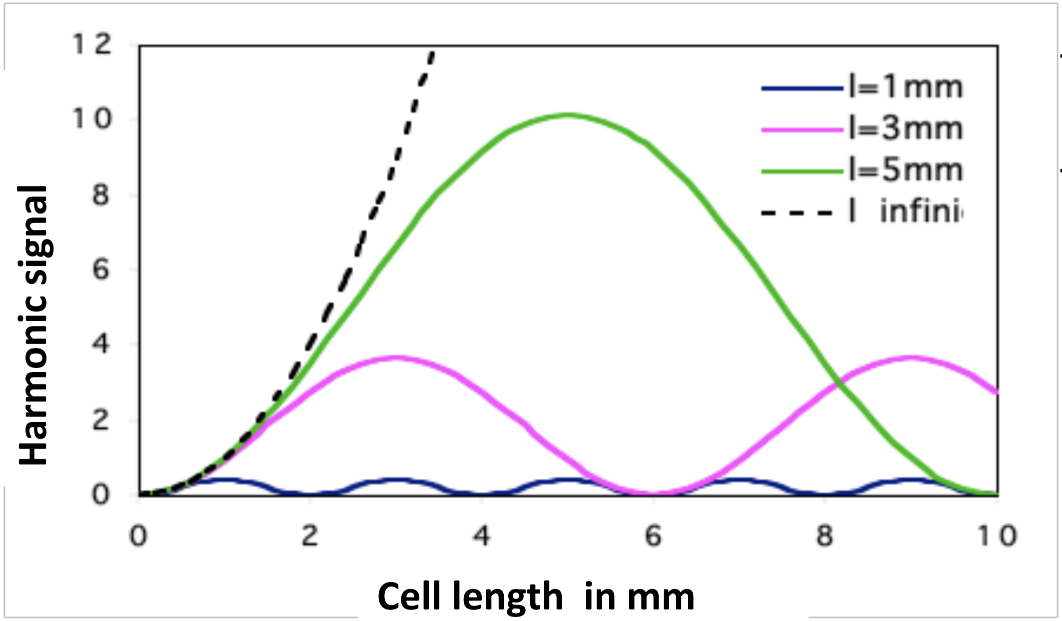
Each individual harmonic dipole is driven by the laser with its own amplitude and phase
It then propagates until the end of the medium with some delay
The total number of photons produced is given by the coherent sum of all dipoles
->there are constructive or destructive interferences following the phases between the dipoles

$$\nabla^2 E(\omega) + n^2(\omega)\frac{\omega^2}{c^2}E(\omega) = \frac{-\omega^2}{\varepsilon c^2}P^{NL}(\omega)$$

$$E_q \propto \int_0^{l_{med}} \rho |d_q(z)| e^{i\phi(z)} dz$$ with $$\phi(z) = (k_q - qk_{IR})z - \phi_{at,k}$$

Macroscopic aspect of HHG:

If $\varphi(z)$ *is linear*$=\delta k.z$
We introduce $l_{coh}=\pi/\delta k$



Phase matching means:
L_{coh} infinite

The signal growth is quadratic with
The number of emitters (Pressure)

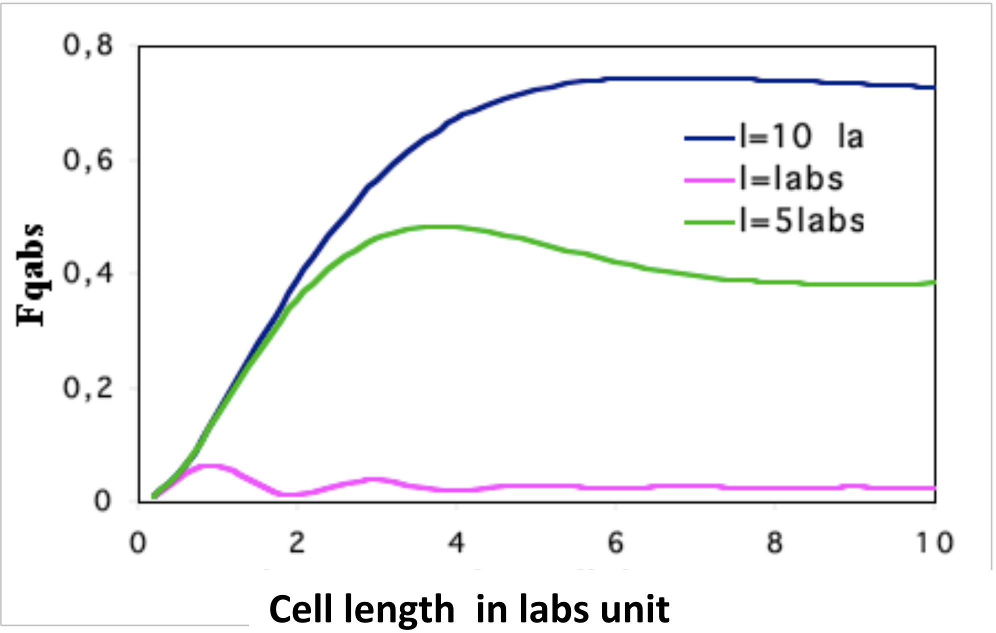
This is the sign of coherent effect

The absorption limit:

$$N_{out}(t) \propto \left| \int_0^{l_{med}} |dq(z)| \exp\left(\frac{z-l_{med}}{2l_{abs}}\right) \exp(i\,\varphi(t,z))\,dz \right|^2$$

L_{abs} varies as 1/P
$T=e^{-l/l_{abs}}$

Saturation of the signal is obtained
When: $l_{med}>3l_{abs}$ and $l_{coh}>5l_{abs}$



Constant et al., PRL 82,1668 (1999)

What is the origin of dephasing?

- Role of the Gouy phase term, balance with

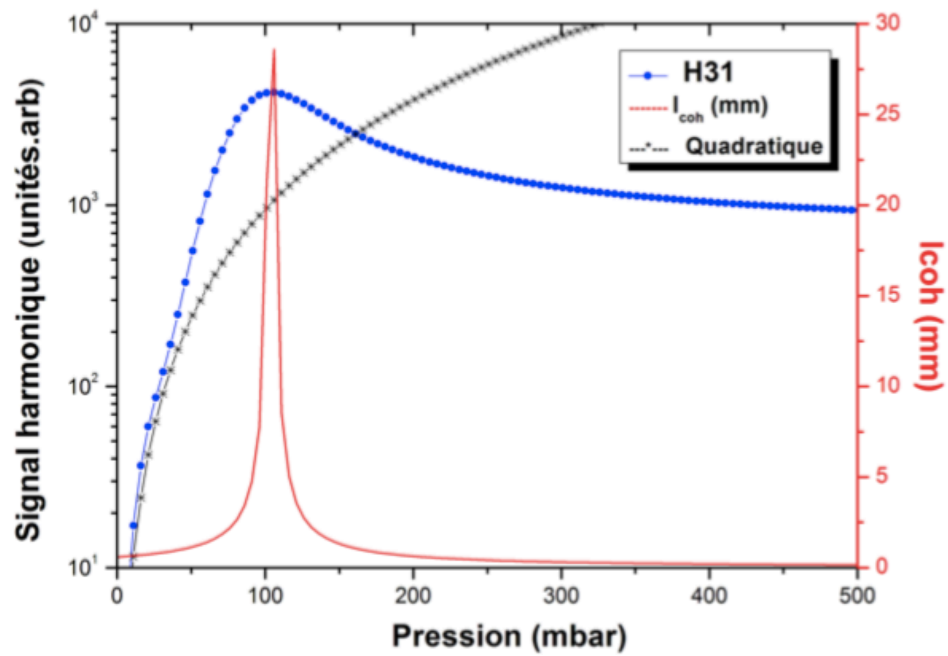
Atomic dispersion if ionization is negligible

$$l_{\mathrm{coh}}(\eta)=\pi/q/1/z\downarrow 0\ \ -10\uparrow -3\ P\,(1.66-7366\eta)$$

- If the ionization

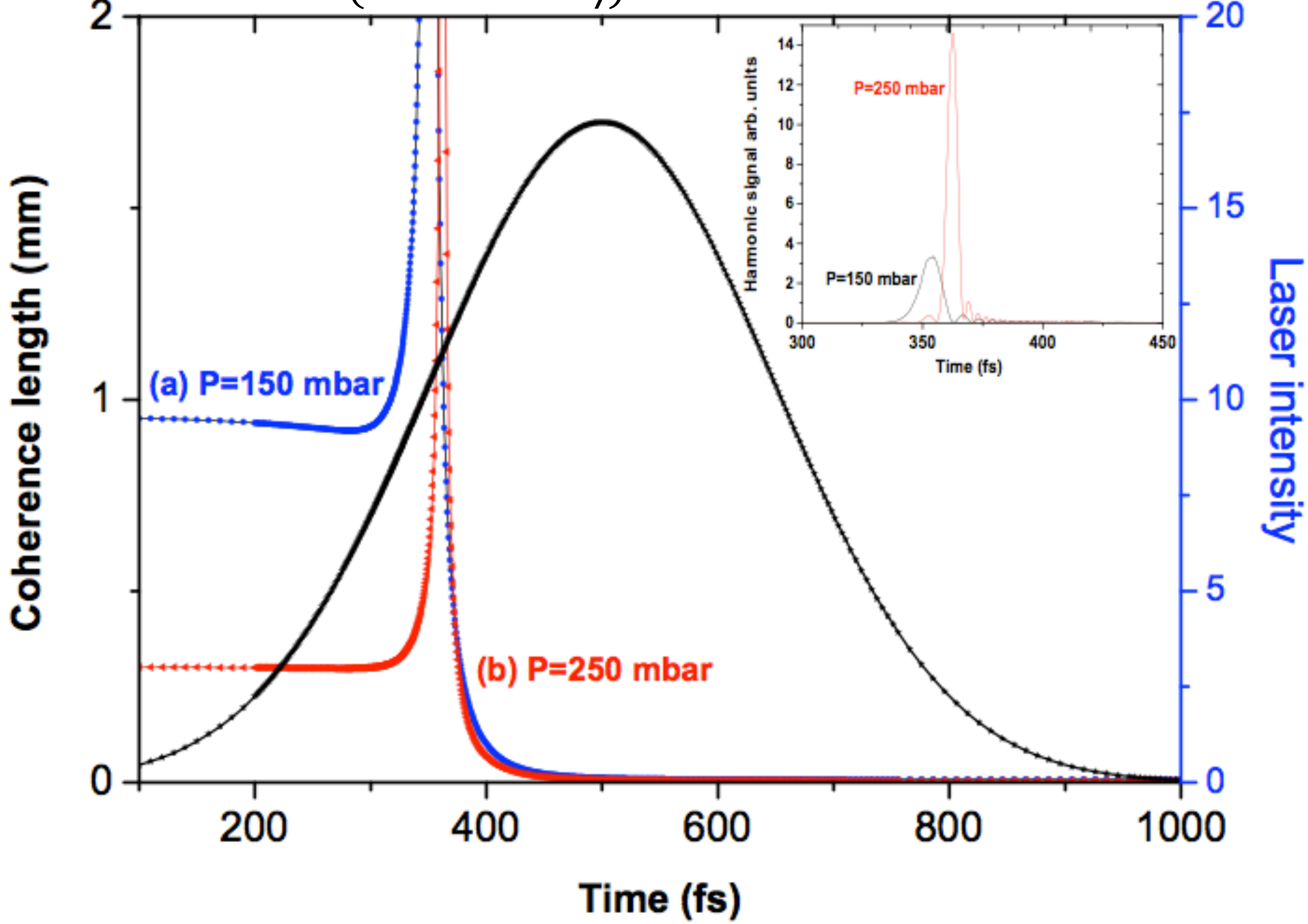
Increases

Then, $n=1-n\downarrow e\,/2n\downarrow c$



The coherence length is time dependant!

$l_{\mathrm{coh}}(\eta)=\pi/q/1/z\downarrow 0 \;\; -10\uparrow -3$
$P\,(1.66-7366\eta)$



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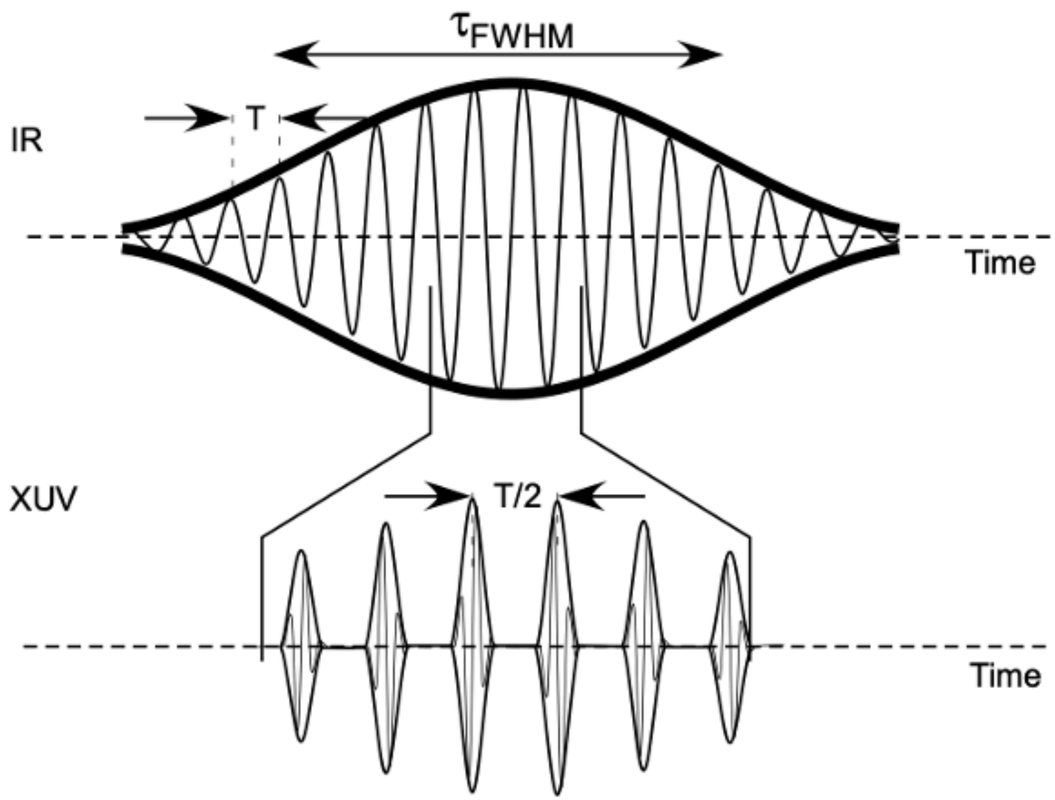
Origin of the attosecond structure for HHG :



The wider the spectrum, the shorter the pulse envelope

Origin of the attosecond structure for HHG :

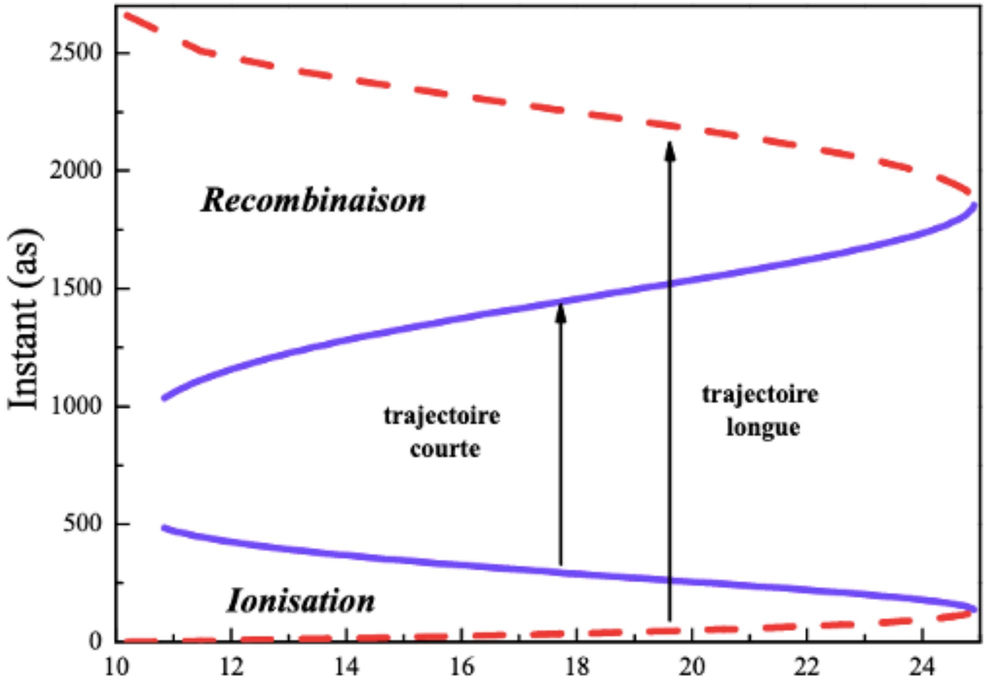


Figure 16: Ionization and recombination times as a function of the harmonic order for a generation in argon at 1.2 10^{14} W/cm^2, in blue the first quantum path, in red the second one, the cutoff is clearly visible (Ph.D. thesis of Yann Mairesse).

How to measure the pulse duration: The Rabbitt method

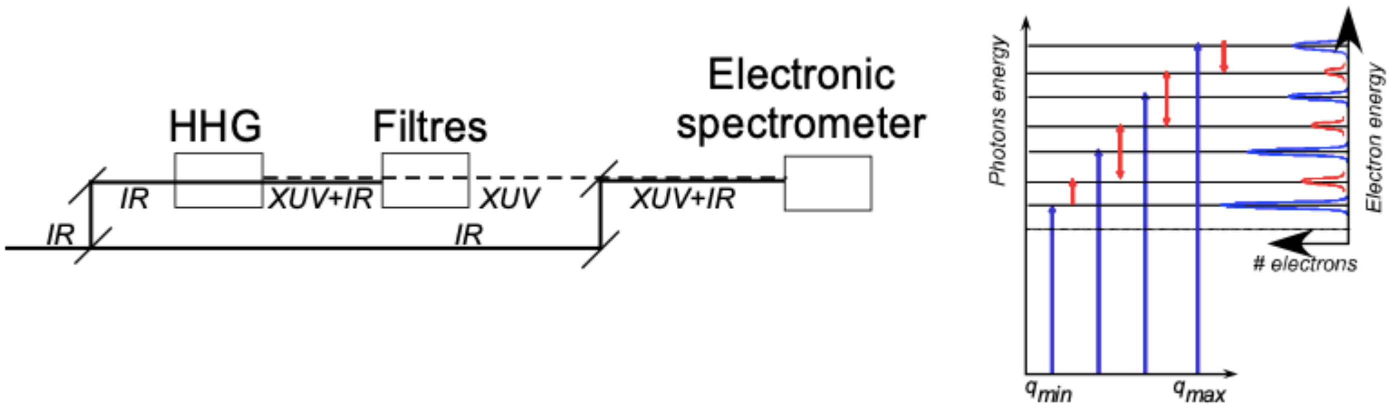


Figure 11: *Principle of the RABBITT technique. (left) The laser IR beam is split into two parts: one used for HHG, the other for dressing. Filters are used to remove the remaining IR after HHG. (Right) Cartoon of the spectra obtained. Sidebands show up right in between odd harmonics electrons coming from XUV photoionization by the attosecond pulse (taken from a review paper by Thierry Ruchon).*

$$2\omega\tau + \Psi_{q'+1} - \Psi_{q'-1}$$

How to measure the pulse duration: The Rabbitt method

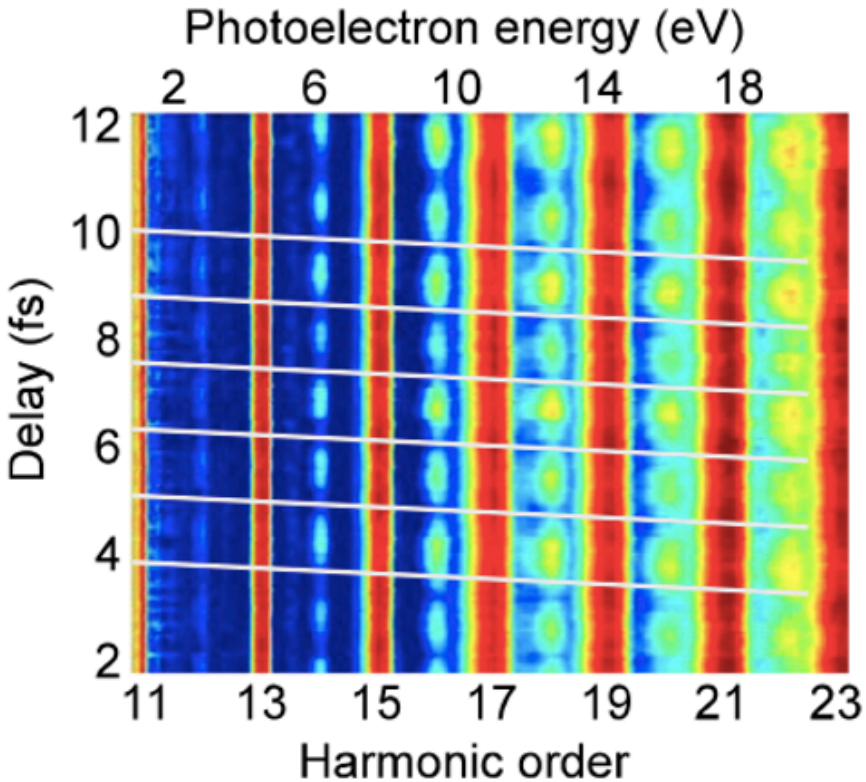
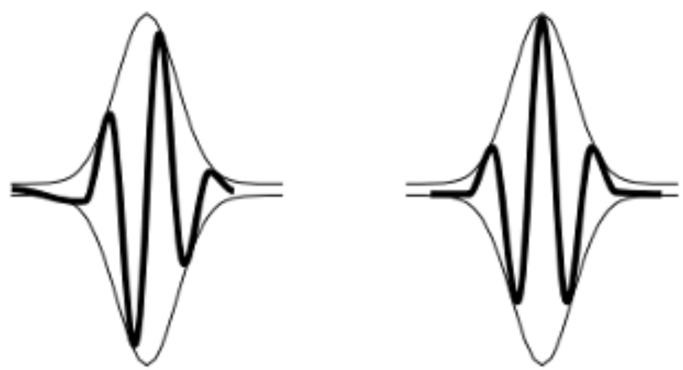


Figure 20: Example of a RABBITT trace taken in Ar for both the generating and detecting gas. The minima in the sidebands vs the delay drift from one sideband to the next signaling a lack of synchronization of the harmonics (superimposed white lines). This figure is taken from a review paper by Th. Ruchon

Single attosecond pulses



$e^{-t^2/\tau^2} \sin \omega_0 t$ $e^{-t^2/\tau^2} \cos \omega_0 t$

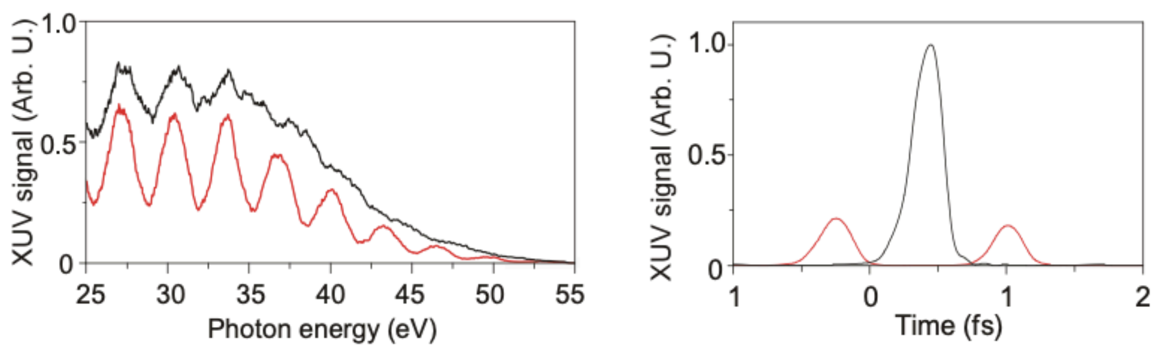


Figure 18: Effect of a change of the CEP on the spectrum (left panel) and corresponding temporal profile (right panel) for an attosecond source driven by a polarization gate field (from [Sola 2006]).

Plan of the lecture:

- The most characteristic features of HHG, historical aspects

- How it looks like experimentally?

- The physical origin of the non linear polarization of atoms

- The problem of phase matching

- An ultrashort story of the attosecond structure

- Modern trends in quantum optics

Modern trends in quantum optics

- Until now in the theory of HHG, only the atoms and the electrons were considered quantum, the laser electric field was supposed to be classical.
- This was a good approximation since the field was very high.
- New interesting results appeared when considering the quantum aspect of light.

Still theoretical prediction

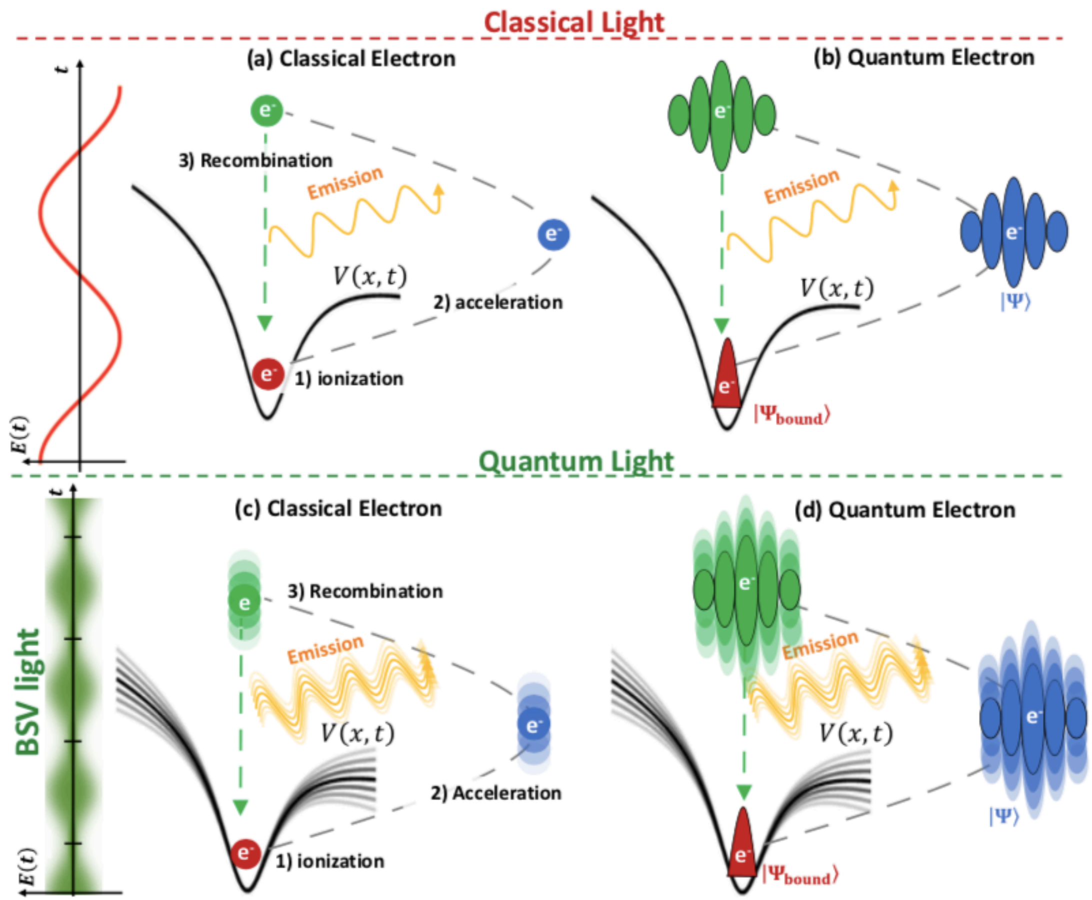
High harmonic generation driven by quantum light

Alexey Gorlach†[1], Matan Even Tzur†[2], Michael Birk[1,2], Michael Krüger[2], Nicholas Rivera[3], Oren Cohen[2] and Ido Kaminer[1]

[1] Department of Electrical and Computer Engineering and Solid State Institute, Technion – Israel Institute of Technology, 32000 Haifa, Israel

[2] Department of Physics and Solid State Institute, Technion – Israel Institute of Technology, 32000 Haifa, Israel

[3] Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02139, USA



Still theoretical prediction

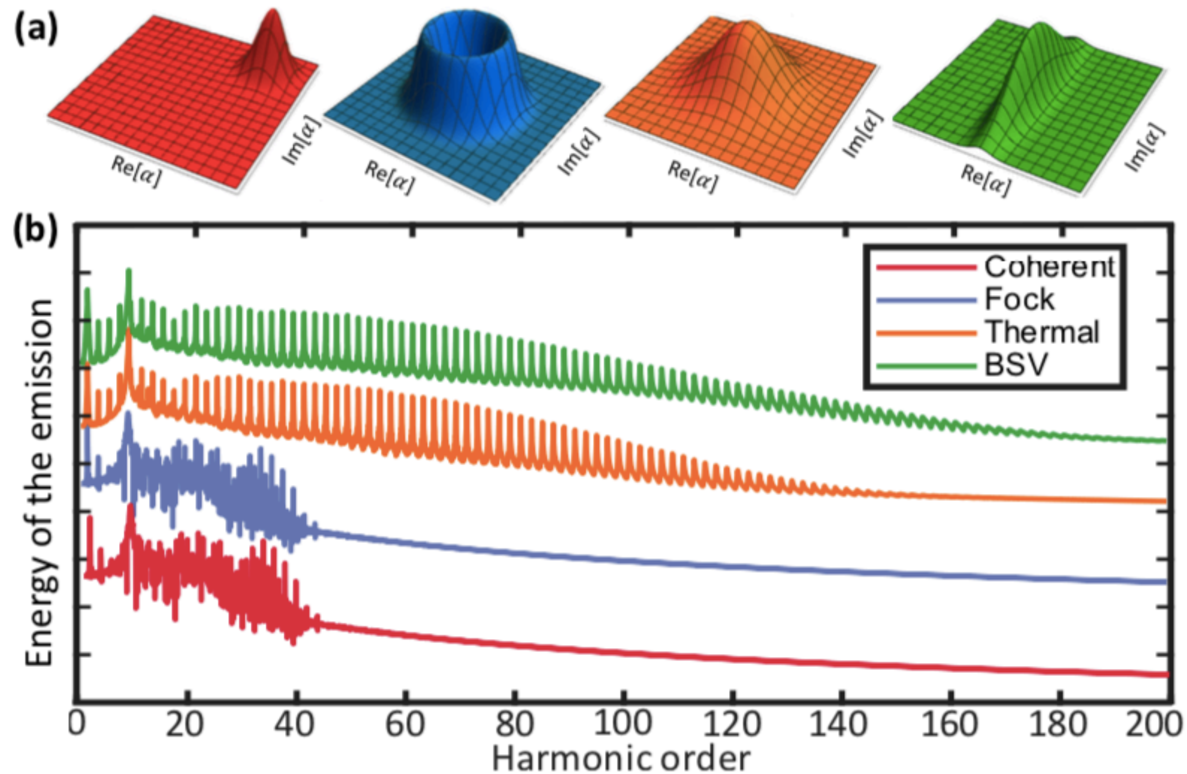


Fig 3. The effect of quantum photon statistics on the spectrum of high harmonic generation. **(a)** Husimi distribution $Q(\alpha)$ of the light state, which is approximately sufficient to determine the entire HHG emission spectrum. The Husimi distribution is displayed here for a coherent state (red), Fock state (blue), thermal state (orange), and bright squeezed vacuum state (green) **(b)** The high harmonic spectra in logarithmic scale for the coherent, Fock, thermal and bright squeezed vacuum states. The intensities, frequencies, and polarizations for all the

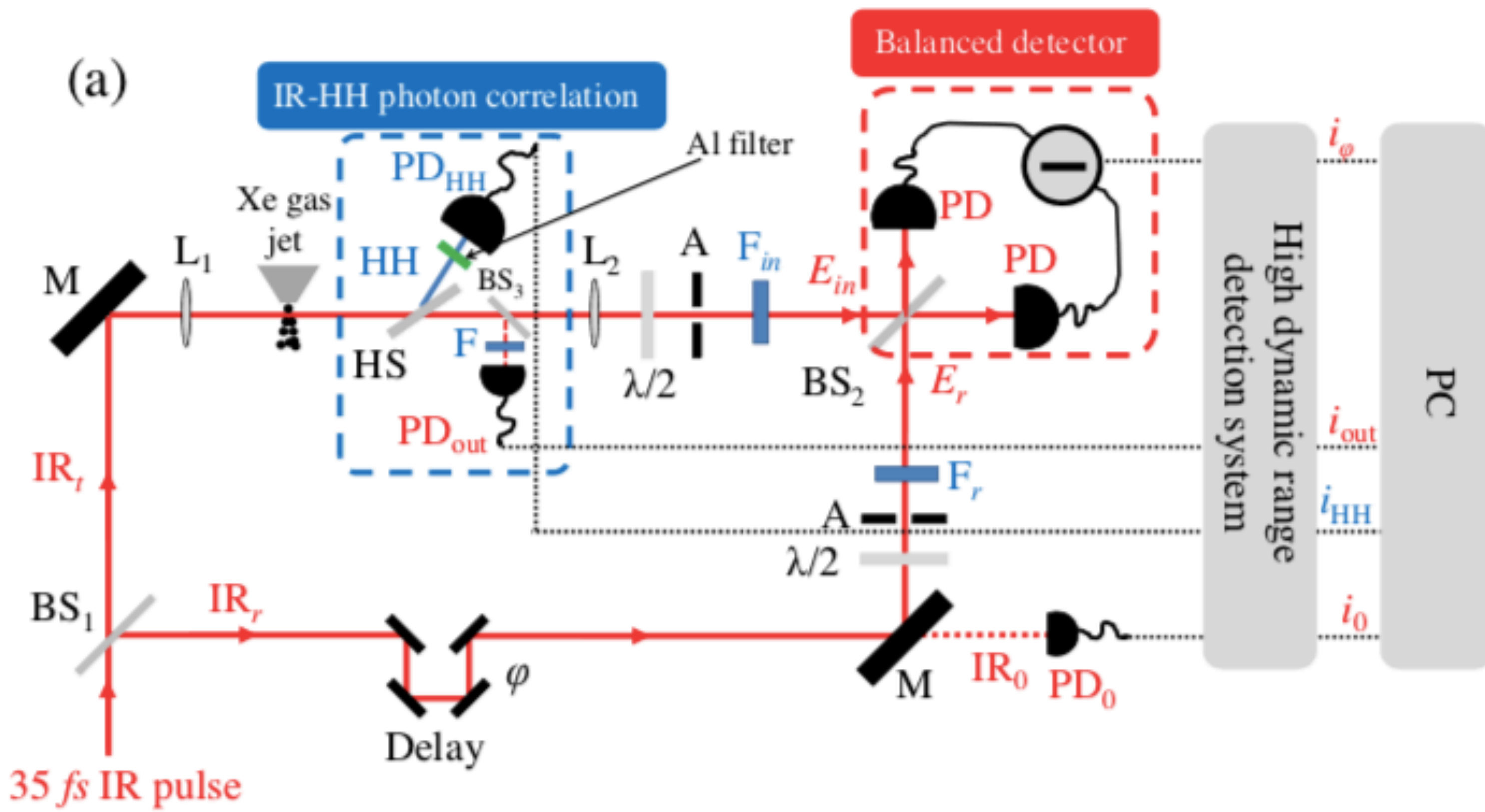
Generation of light vortices

- This is a very active field of research: in the quantum view each photon is supposed to carry an orbital momentum of $l\hbar$, in the classical view the electric field spatial phase writes: $e^{il\theta}$
- In HHG: $l_q = q l_1$
- An interplay may exist with the « spin » of the photons (the polarization) or with atoms interacting with this kind of light

Generation of optical Schrödinger cat states in intense laser-matter interactions

M. Lewenstein [1,2*], M.F. Ciappina [1,3,4,5], E. Pisanty [1,6], J. Rivera-Dean [1], P. Stammer [1,6], Th. Lamprou [7,8], and P. Tzallas [7,9*]

[1] ICFO – Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain



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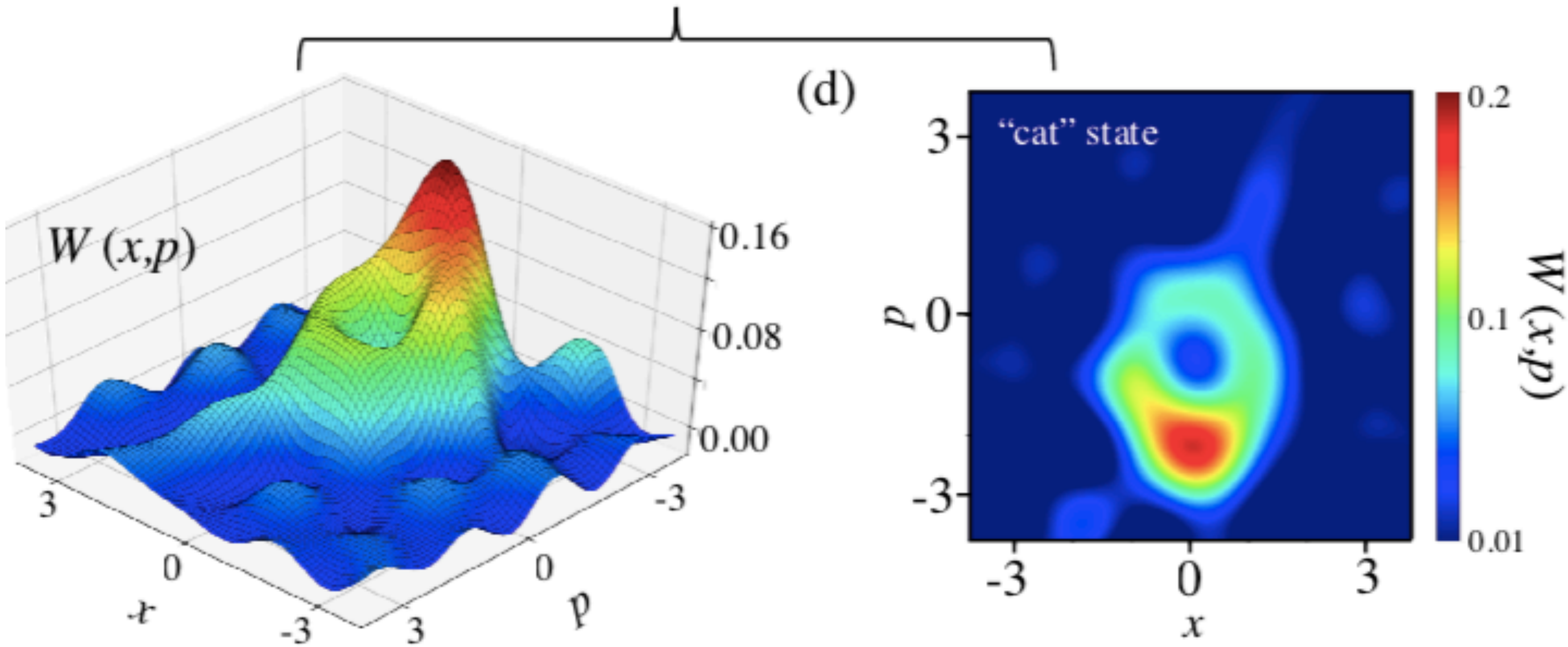

A superposition of several different states of light: not compatible classically

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