ULTRACOLD ATOMS IN OPTICAL CAVITIES

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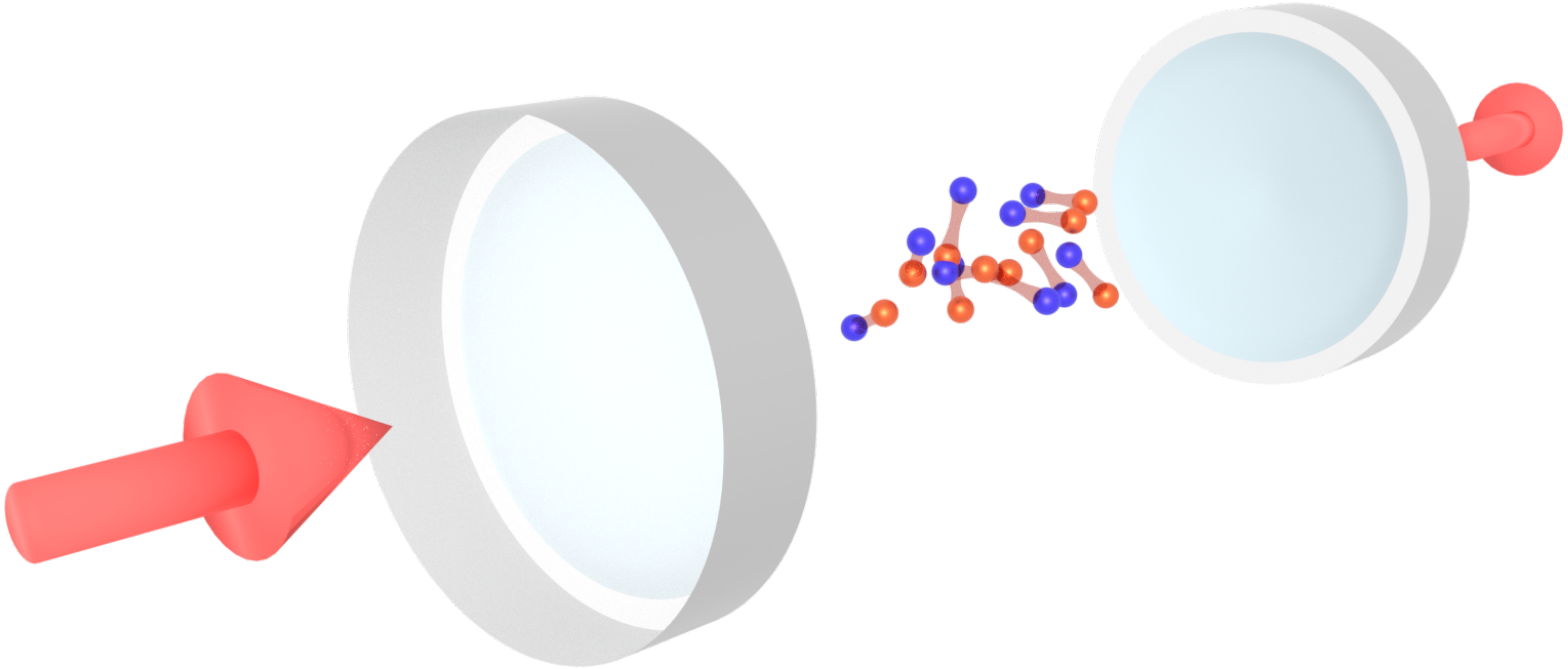
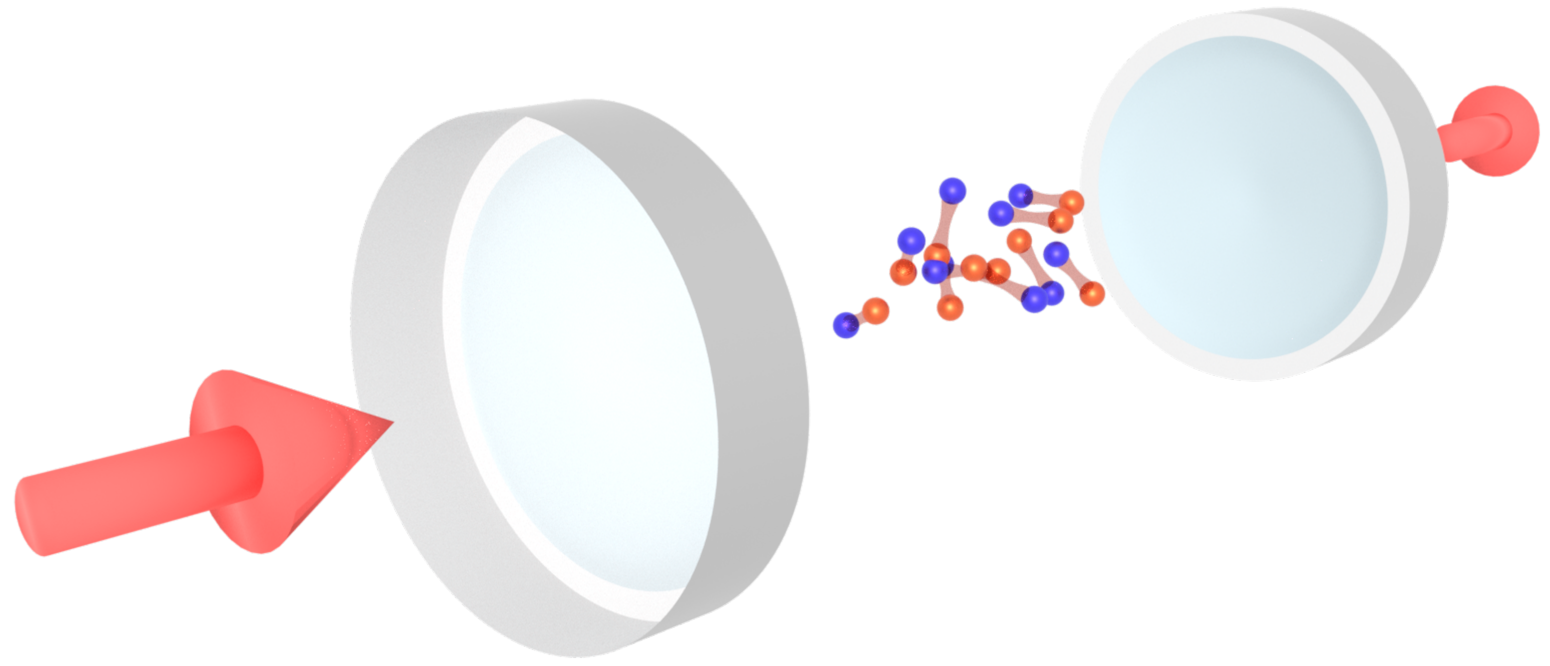
Università degli Studi di Firenze, EPFL Lausanne



11/05/2023

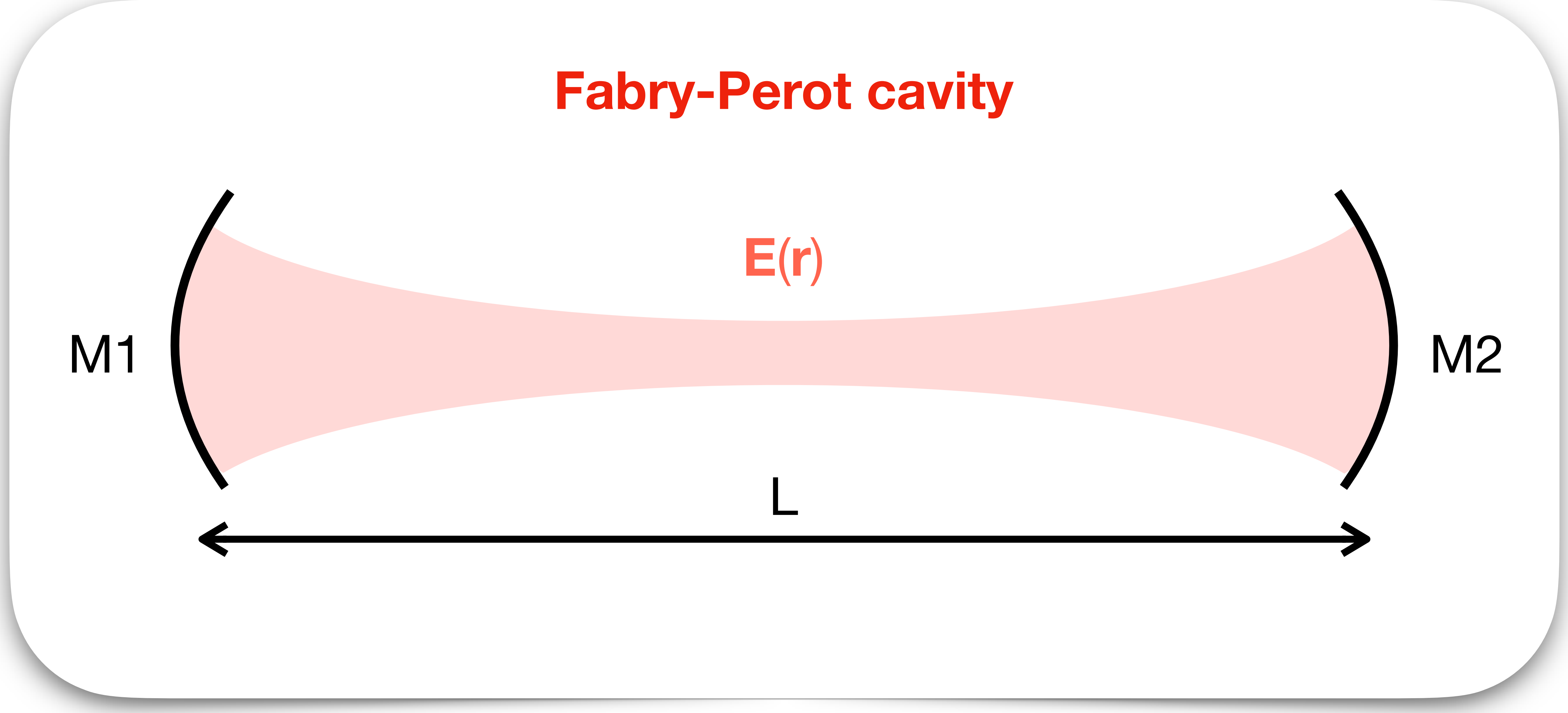
Quantum NEST - Trieste

OUTLINE OF THIS LECTURE



- Overview of optical cavities
- Strong light-matter coupling: collective effect of many atoms in a cavity
- Cavity-mediated long-range interactions

OPTICAL CAVITIES



1 Frequency of the EM field inside the cavity

After a roundtrip, the light should have acquired a phase of 2π (or an integer multiple), so to have constructive interference

$$\lambda = 2L \times p \quad \rightarrow \quad p \in \mathbb{N}$$

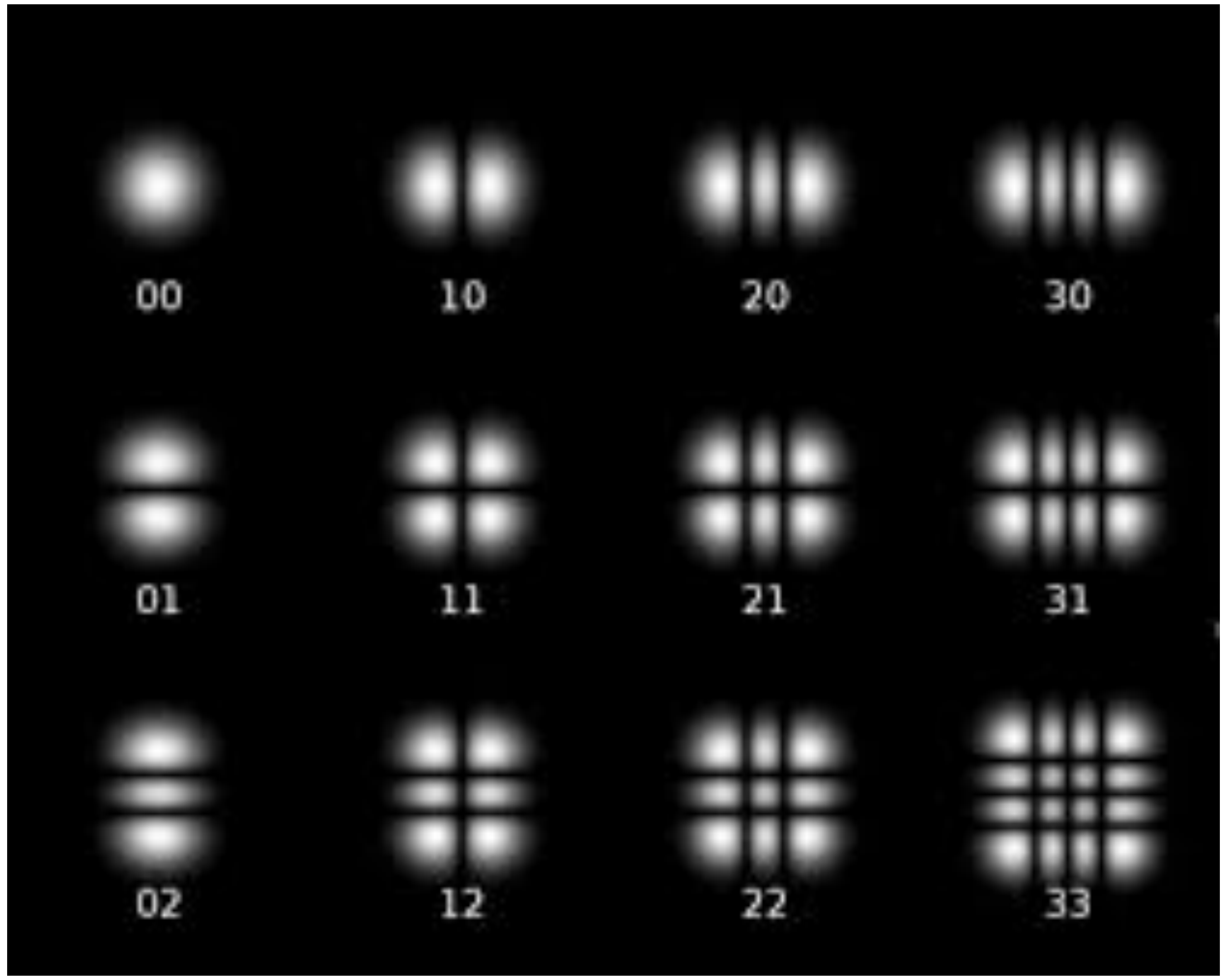
Longitudinal modes

2 Modes of the EM field inside the cavity

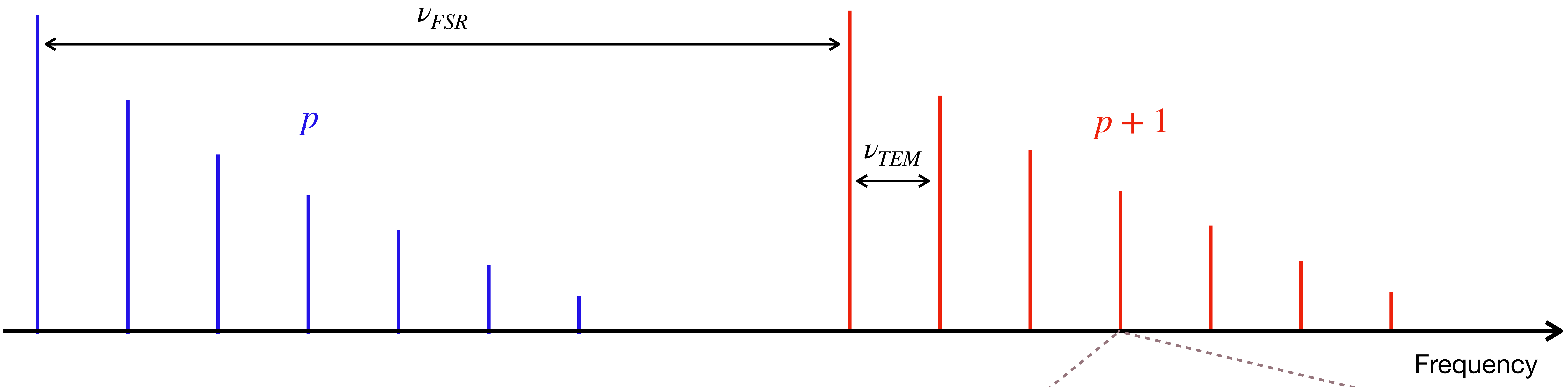
Solve the Helmholtz equation: $\nabla^2 \mathbf{E}(\mathbf{r}) + k\,\mathbf{E}(\mathbf{r}) = 0$

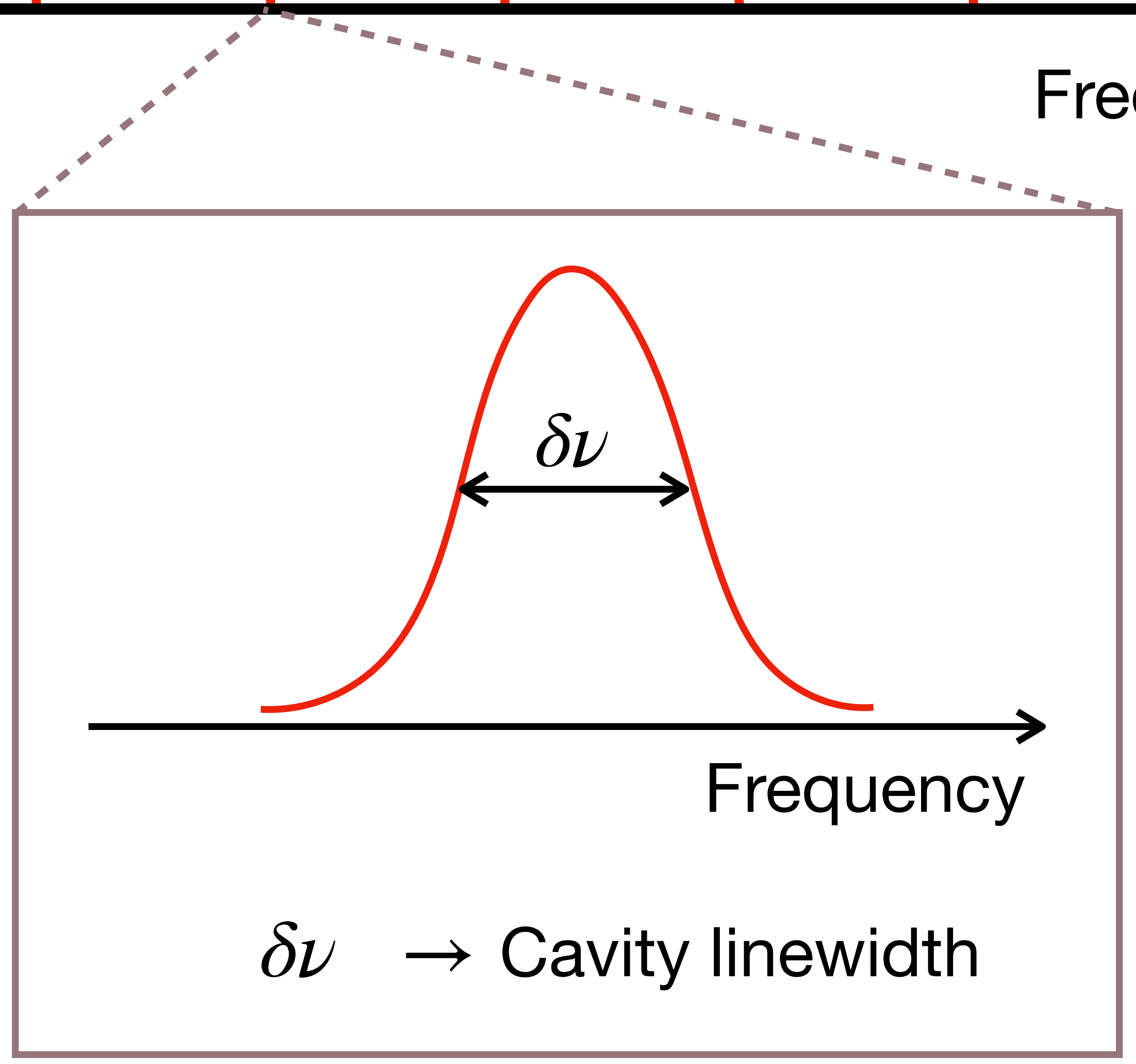
$k = 2\pi/\lambda \rightarrow$ wavevector

Transverse modes: Hermite-Gauss $\text{TEM}_{n,m}$



OPTICAL CAVITY SPECTRUM



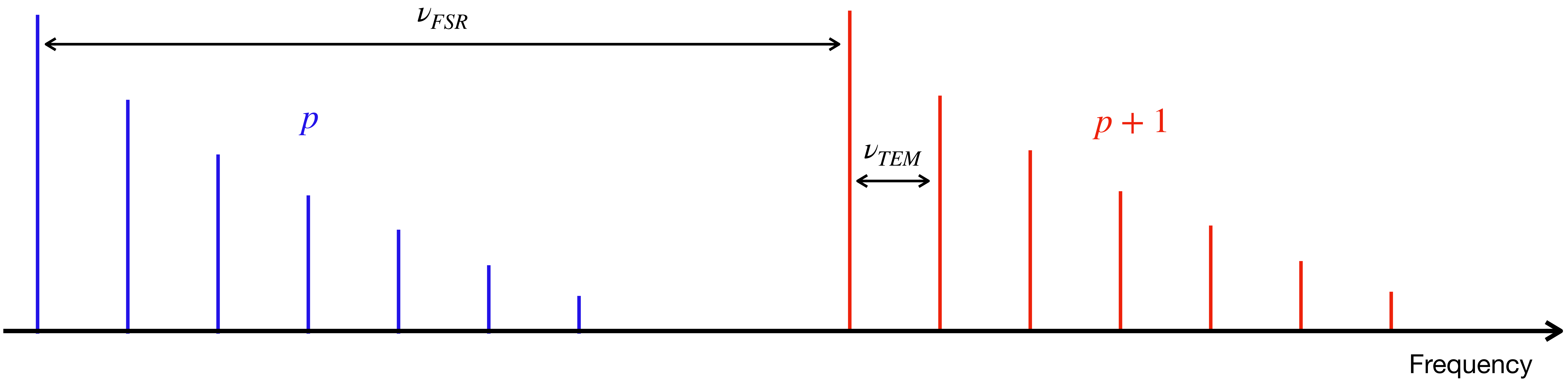


$\nu_{FSR} = c/2L \quad \rightarrow$ Free Spectral Range

$\nu_{TEM} = c/2\pi L \arccos(1 - L/R_c) \quad \rightarrow$ Transverse mode spacing

$R_c \quad \rightarrow$ curvature radius of the mirrors

OPTICAL CAVITY SPECTRUM



$\nu_{FSR} = c/2L$ → Free Spectral Range

$\nu_{TEM} = c/2\pi L \arccos(1 - L/R_c)$ → Transverse mode spacing

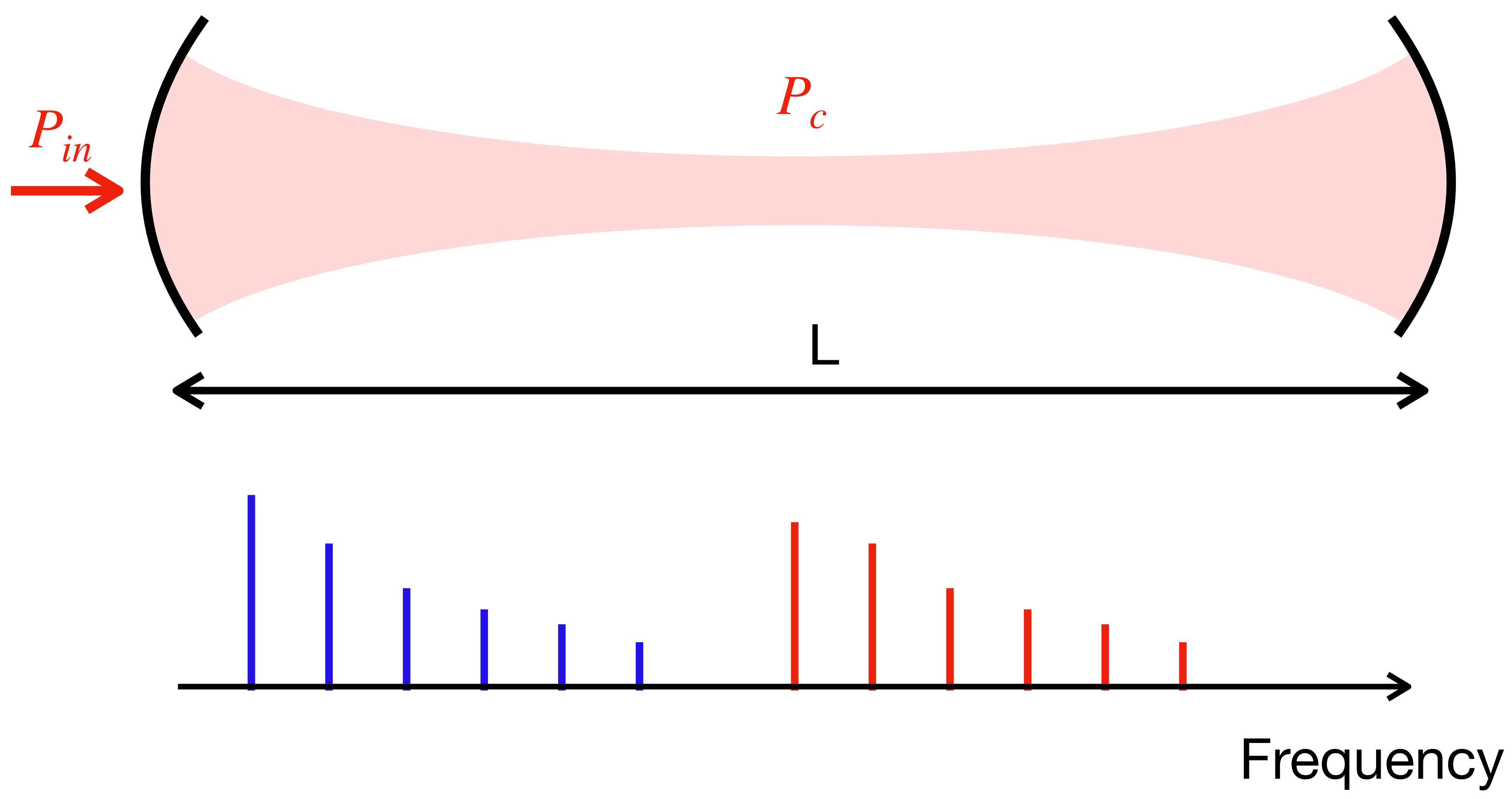
R_c → curvature radius of the mirrors

Cavity Locking

Fix L of the cavity such that it is resonant with a reference frequency:

- Laser
- Spectroscopy
- Other cavity

WHY OPTICAL CAVITIES ARE INTERESTING?



1. Inside an optical cavity exist a highly monochromatic EM field at high power.

$$\frac{P_c}{P_{in}} = \frac{4}{\pi}\mathscr{F}$$

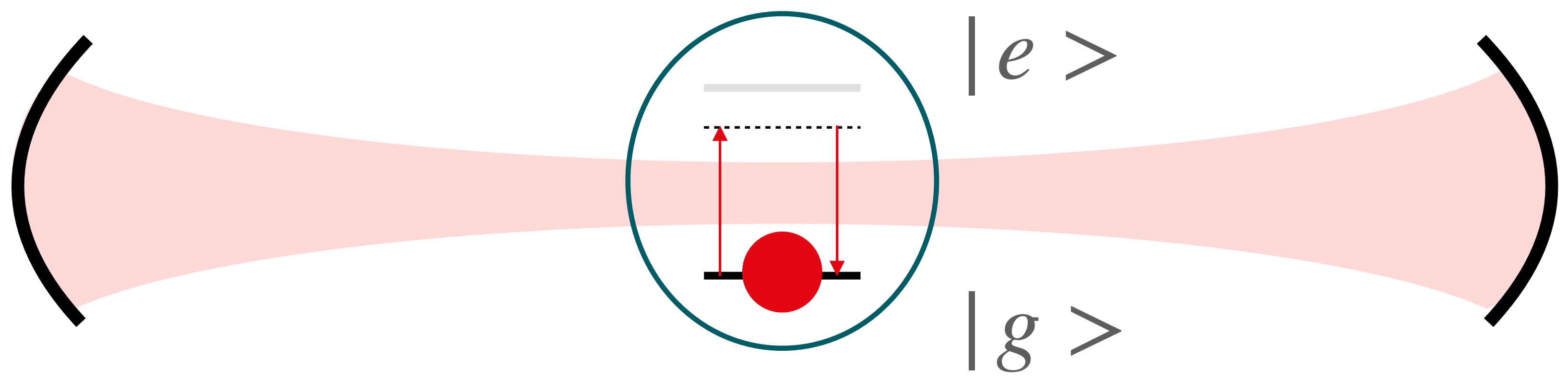
$$\mathscr{F} = \frac{\nu_{FSR}}{2\,\delta\nu} \sim 10^3 - 10^4 \rightarrow \text{Finesse of the cavity}$$

Strong light-matter interactions

2. The cavity modifies the density of states of the atoms, affecting both the absorption/emission and the scattering properties, even when no photons populate the cavity mode.

Cavity-mediated long-range interactions

SINGLE ATOM IN AN OPTICAL CAVITY



$E_e - E_g = \hbar\omega_A$ → atomic resonance frequency

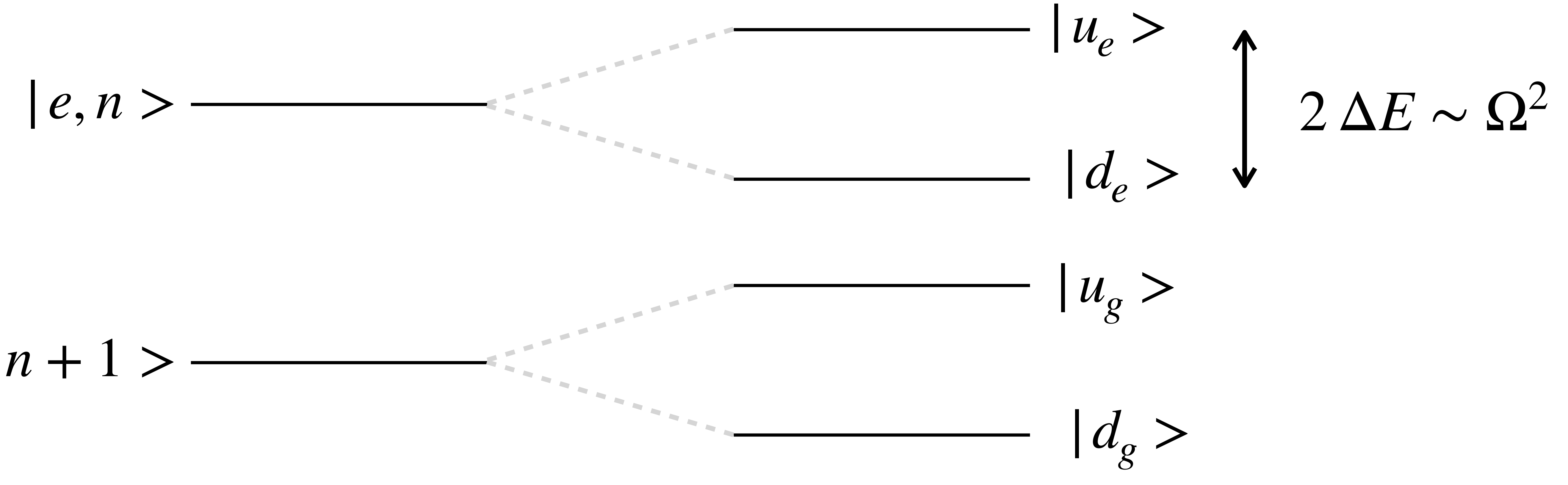
$\Omega = \dfrac{2|d_{ge}||E|}{\hbar}$ → Rabi frequency of the coupling

$a, a^\dagger$ → annihilation/creation operators of photons

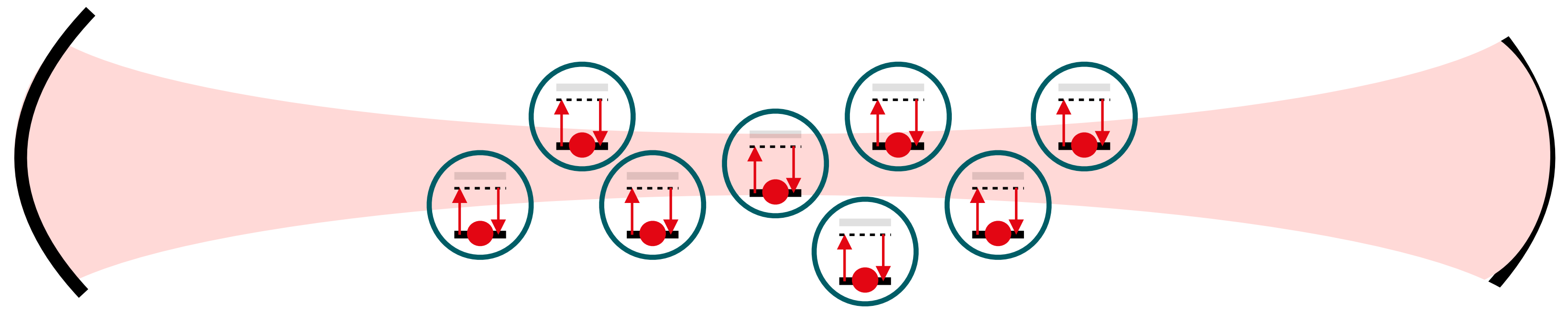
$$\mathscr{H} = \overbrace{\hbar\omega_c a^\dagger a}^{\text{Cavity}} + \overbrace{\hbar\omega_A |e><e|}^{\text{Atom}} + \overbrace{\hbar\Omega\left(|e><g| + |g><e|\right)\left(a^\dagger + a\right)}^{\text{Interactions}}$$

Strong coupling = big Ω:

Dressed states: hybrid states of atom and photons



MANY ATOMS IN AN OPTICAL CAVITY



N atoms of two levels $|g>$ and $|e>$

$\omega_c \rightarrow$ cavity frequency

$\omega_A \rightarrow$ atom resonance frequency

$a, a^\dagger \rightarrow$ annihilation/creation operators of photons

Cavity | Atoms | Interactions

$$\mathscr{H} = \underbrace{\hbar\omega_c a^\dagger a}_{\text{Cavity}} + \underbrace{\sum_i \hbar\omega_A |e>_i <e|_i}_{\text{Atoms}} + \underbrace{\hbar\Omega \sum_i \left(|e>_i <g|_i + |g>_i <e|_i\right)\left(a^\dagger + a\right)}_{\text{Interactions}}$$

$J_k = \sum_i \sigma_k^{(i)} \rightarrow$ Collective operators

$\sigma_k^{(i)} \rightarrow$ Pauli matrices of single atoms

$$\mathscr{H} = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_A}{2} J_z + \frac{\hbar\Omega}{2}\left(J_+ + J_-\right)\left(a^\dagger + a\right)$$

MANY ATOMS IN AN OPTICAL CAVITY

$$\mathscr{H} = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_A}{2} J_z + \frac{\hbar\Omega}{2}\left(J_+ + J_-\right)\left(a^\dagger + a\right)$$

The collective operators form an **angular momentum algebra** with $J^2 = \dfrac{3}{4}N$

$$[J_k, J_l] = i\,\epsilon_{klm} J_m \quad , \quad [J^2, J_k] = 0$$

$\{\,|j,m>\,\}$ $\rightarrow$ **Dicke states**

$j = N/2$

$m = -N/2, \ldots, N/2$

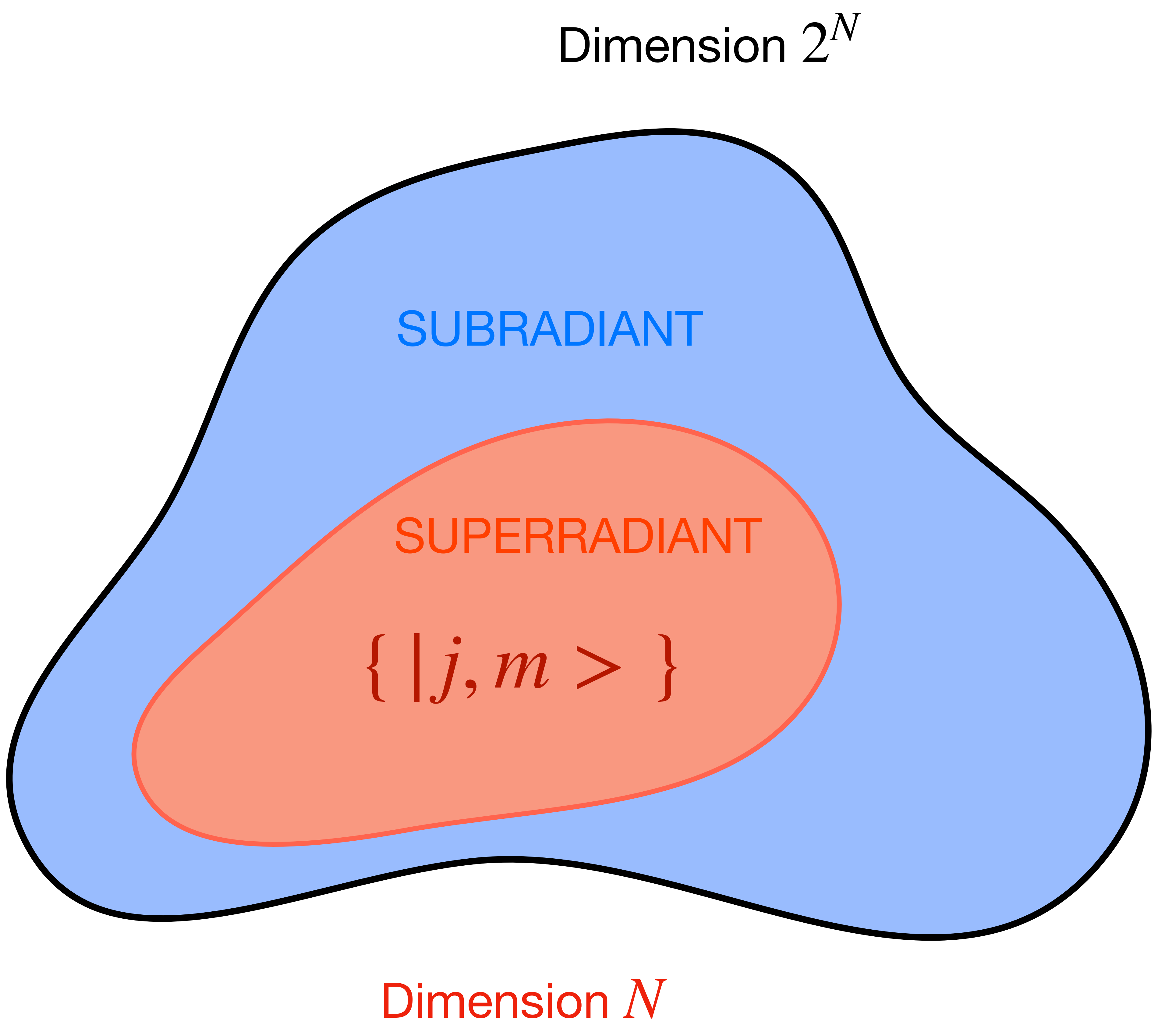
$\longrightarrow$

$J^2\,|j,m> = j(j+1)\,|j,m>$

$J_z\,|j,m> = m\,|j,m>$

$J_\pm\,|j,m> \sim |j,m\pm 1>$

They are eigenstates of the non-interacting Hamiltonian



Ground state: $|N/2, -N/2> = |g, g, \ldots, g>$

1st excited: $|N/2, -N/2+1> = \sum_i |g, g, e_i, \ldots, g>$ Entangled states!

MANY ATOMS IN AN OPTICAL CAVITY

$$\mathcal{H} = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_A}{2} J_z + \frac{\hbar\Omega}{2}\left(J_+ + J_-\right)\left(a^\dagger + a\right)$$

Holstein-Primakoff transformation

$$J_z = b^\dagger b - j$$

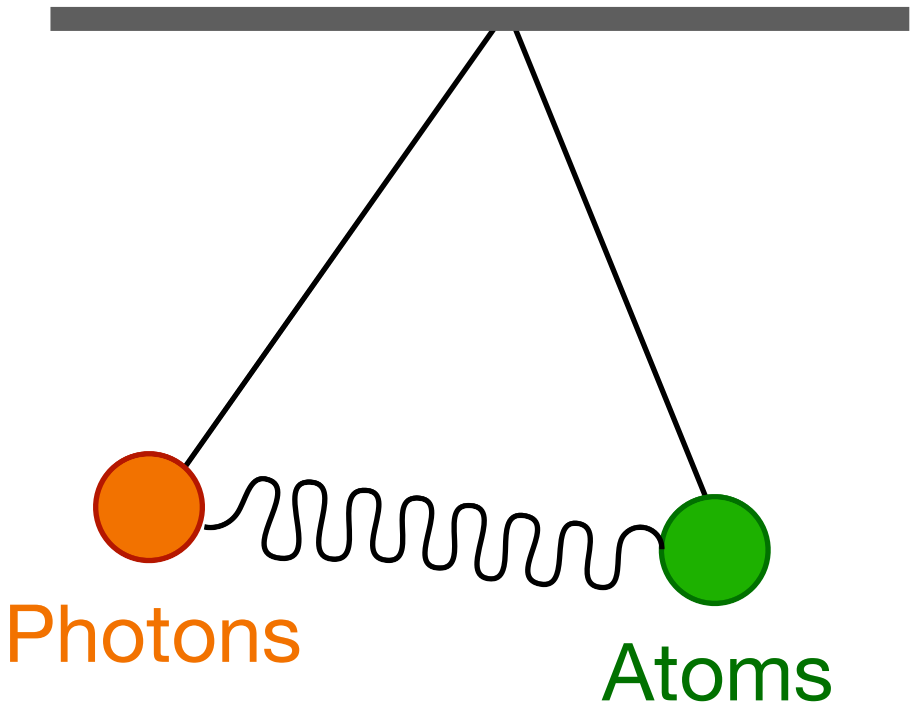
$< b^\dagger b > \ll 1$

$$J_+ = \sqrt{2j}\, b^\dagger \sqrt{1 - \frac{b^\dagger b}{2j}}$$

$b, b^\dagger$ $\rightarrow$ bosonic annihilation/creation operators

$$\mathcal{H} = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_A}{2} b^\dagger b + \sqrt{2j}\,\frac{\hbar\Omega}{2}\left(b^\dagger + b\right)\left(a^\dagger + a\right)$$

$\longrightarrow$ Two coupled harmonic oscillators

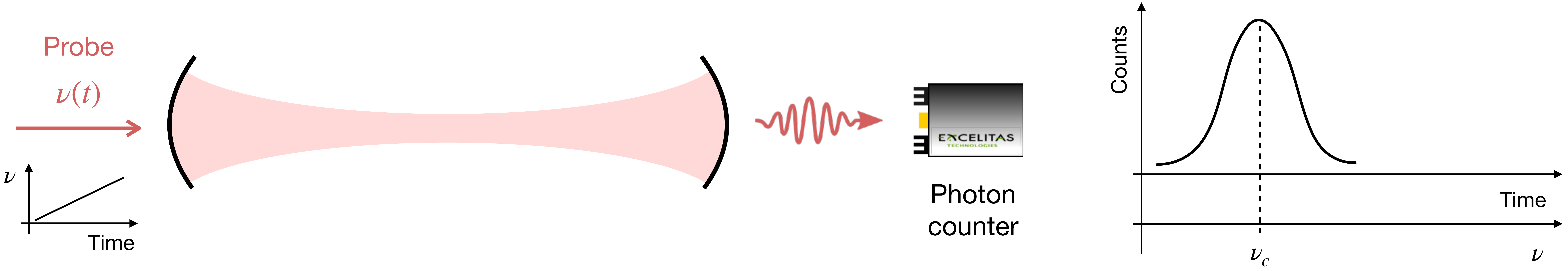


The eigenstates of the Hamiltonian are hybrid states of atoms and photons:

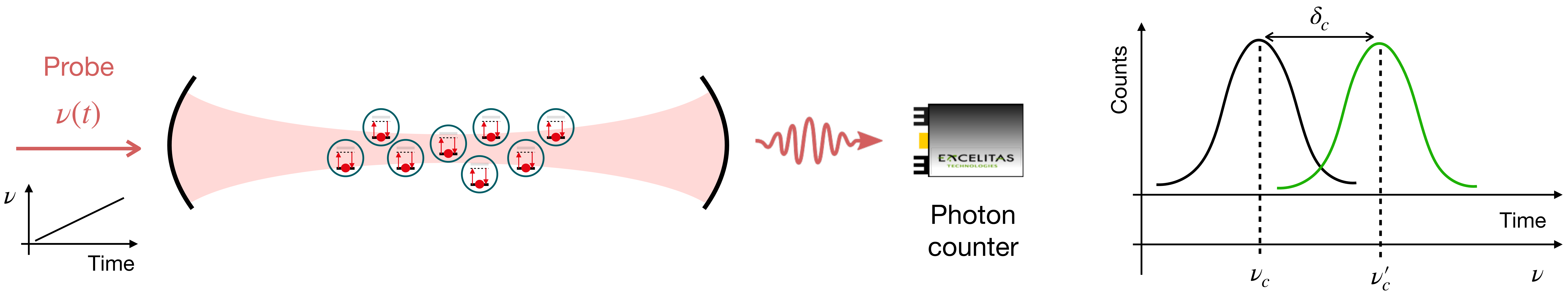
POLARITONS

$$c_\pm = \alpha a \pm \beta b$$

HOW TO PROBE THE POLARITIES: CAVITY TRANSMISSION SPECTROSCOPY



HOW TO PROBE THE POLARITIES: CAVITY TRANSMISSION SPECTROSCOPY



1 **Dispersive regime: $\Delta = \omega_c - \omega_A$ is big**

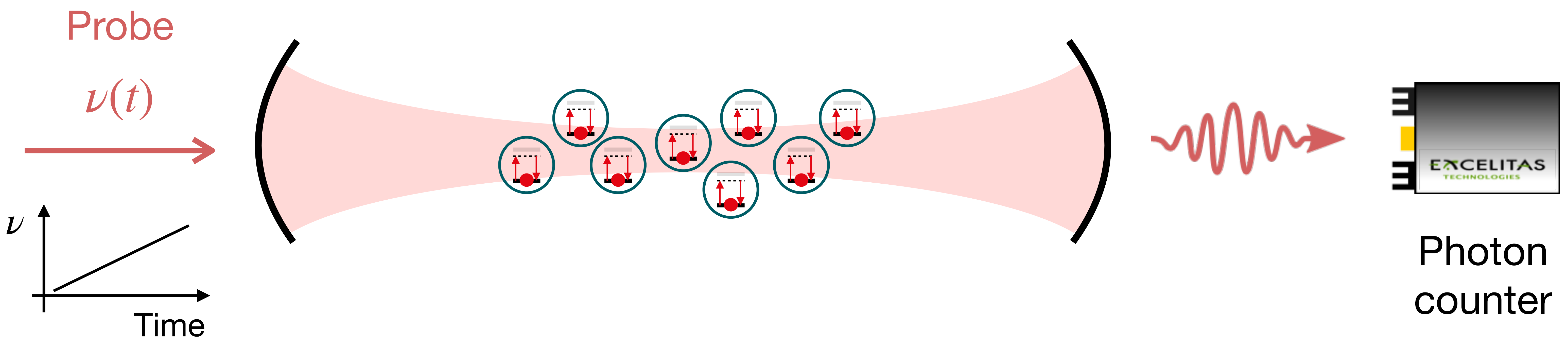
The light traveling in the cavity sees an effective length affected by the presence of the atoms.

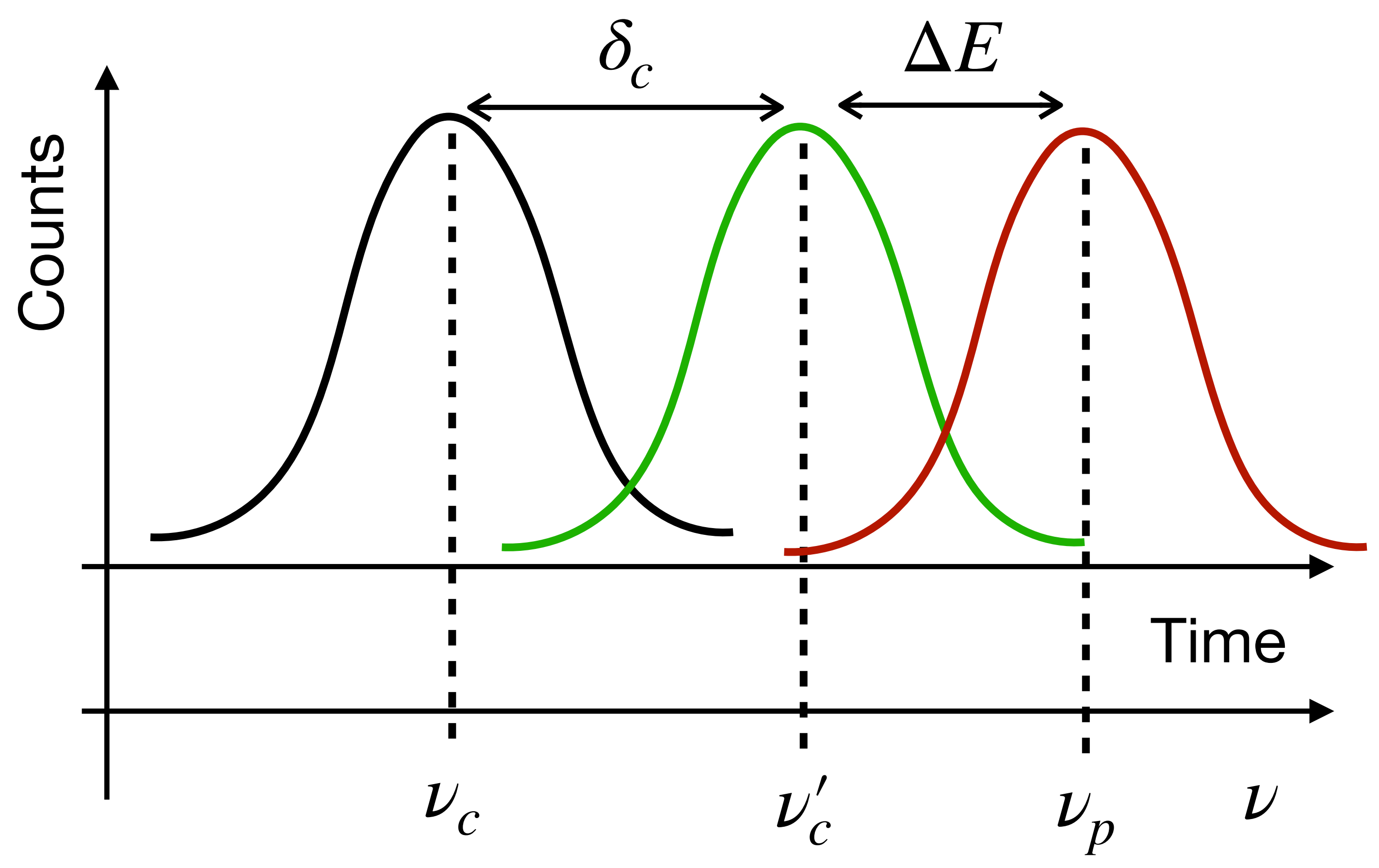
The cavity resonance moves by the dispersive shift:

$$\delta_c = \Omega N/2$$

Cavity transmission spectroscopy provides a **weakly destructive measurement** of the number of atoms

HOW TO PROBE THE POLARITIES: CAVITY TRANSMISSION SPECTROSCOPY



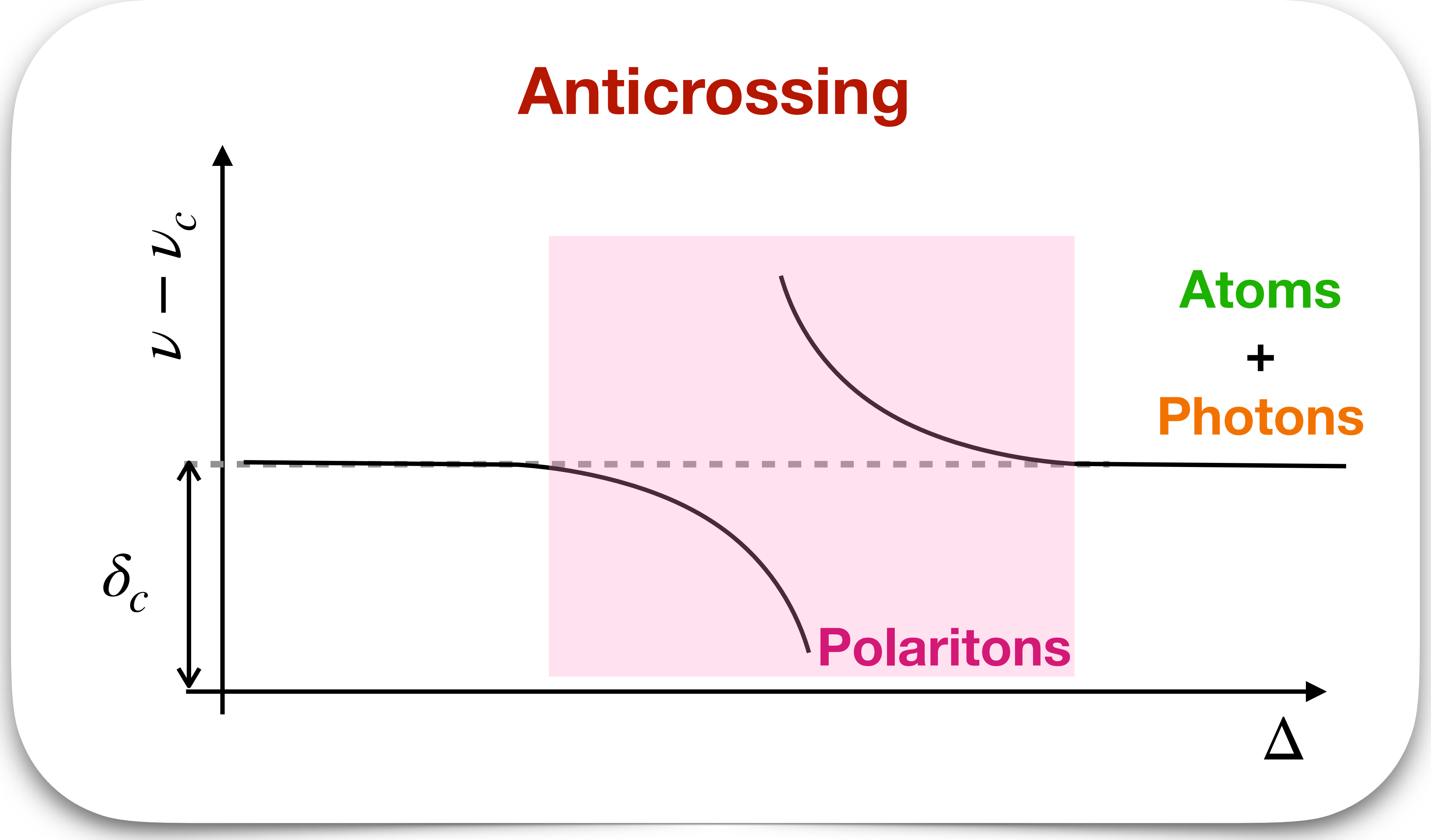


2 Resonant regime: $\Delta = \omega_c - \omega_A$ is small

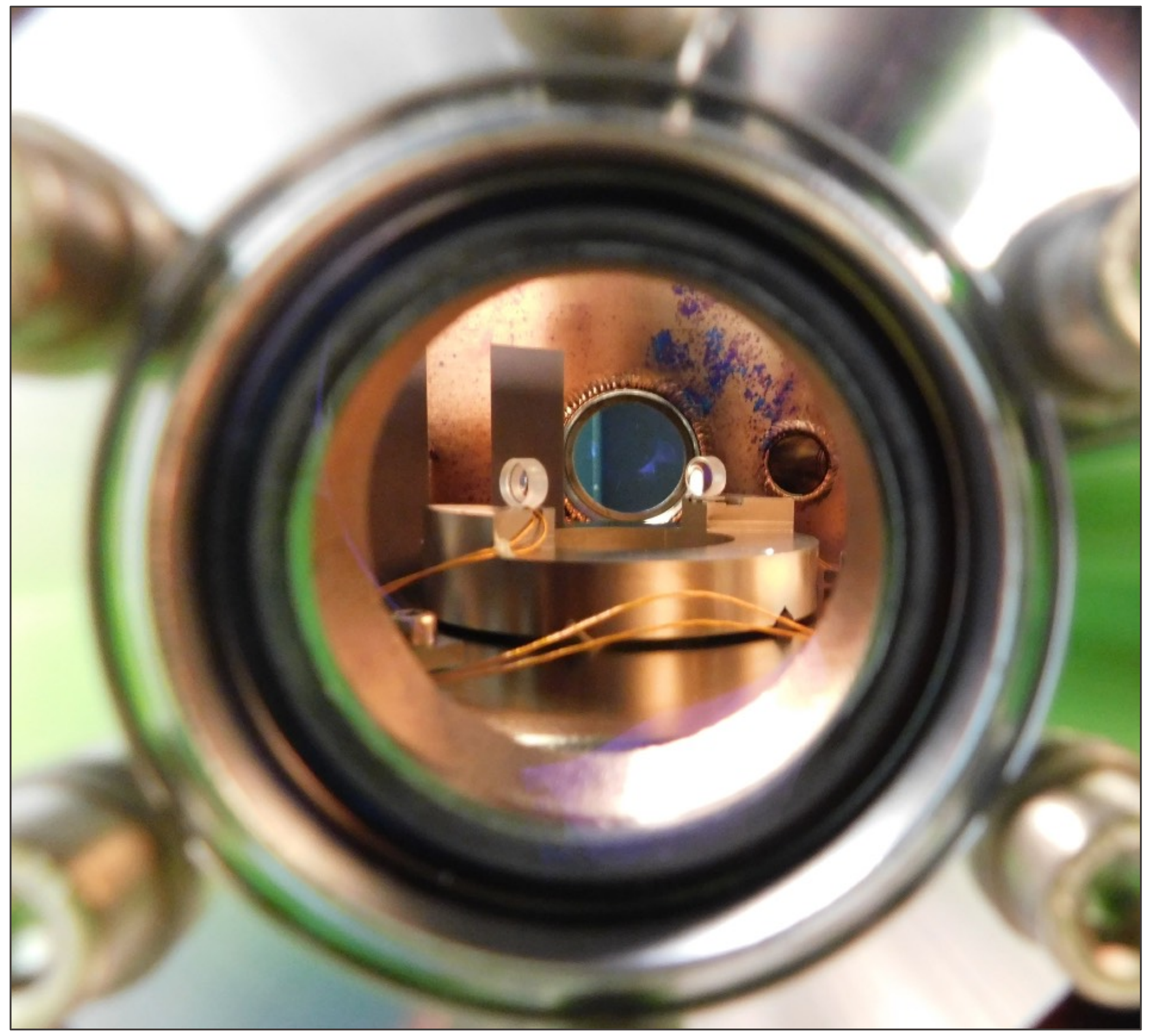
The light traveling in the cavity is strongly-coupled with the atoms: polaritons.

The cavity resonance moves by the polariton energy:

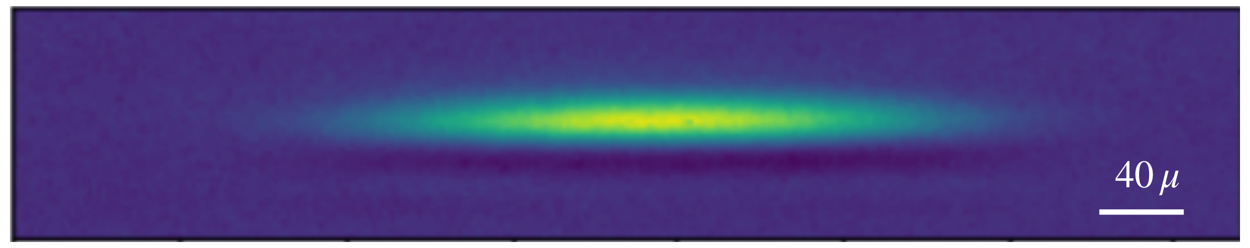
$$\Delta E \sim \Omega \sqrt{N}$$



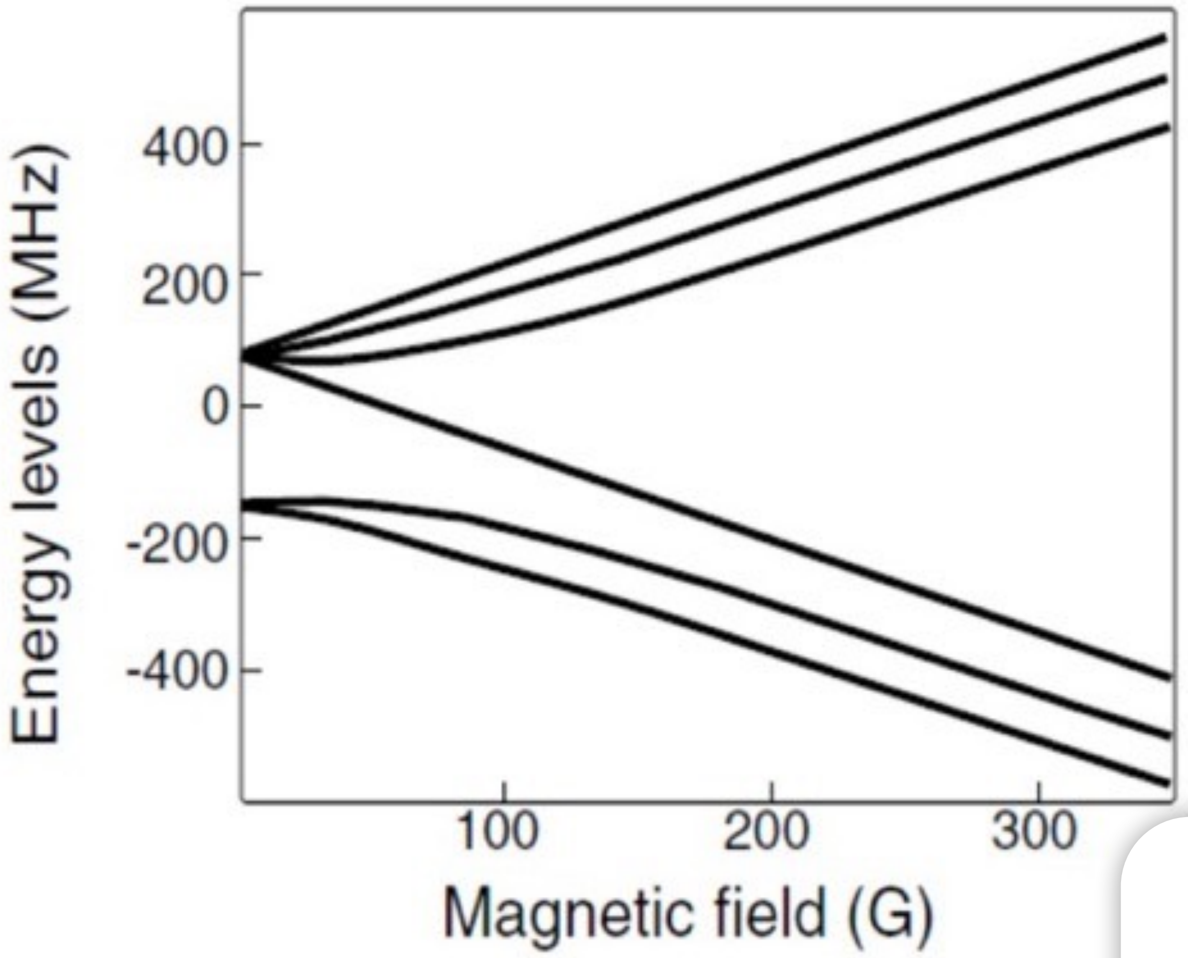
CAVITY TRANSMISSION SPECTROSCOPY ON ULTRACOLD FERMIONS

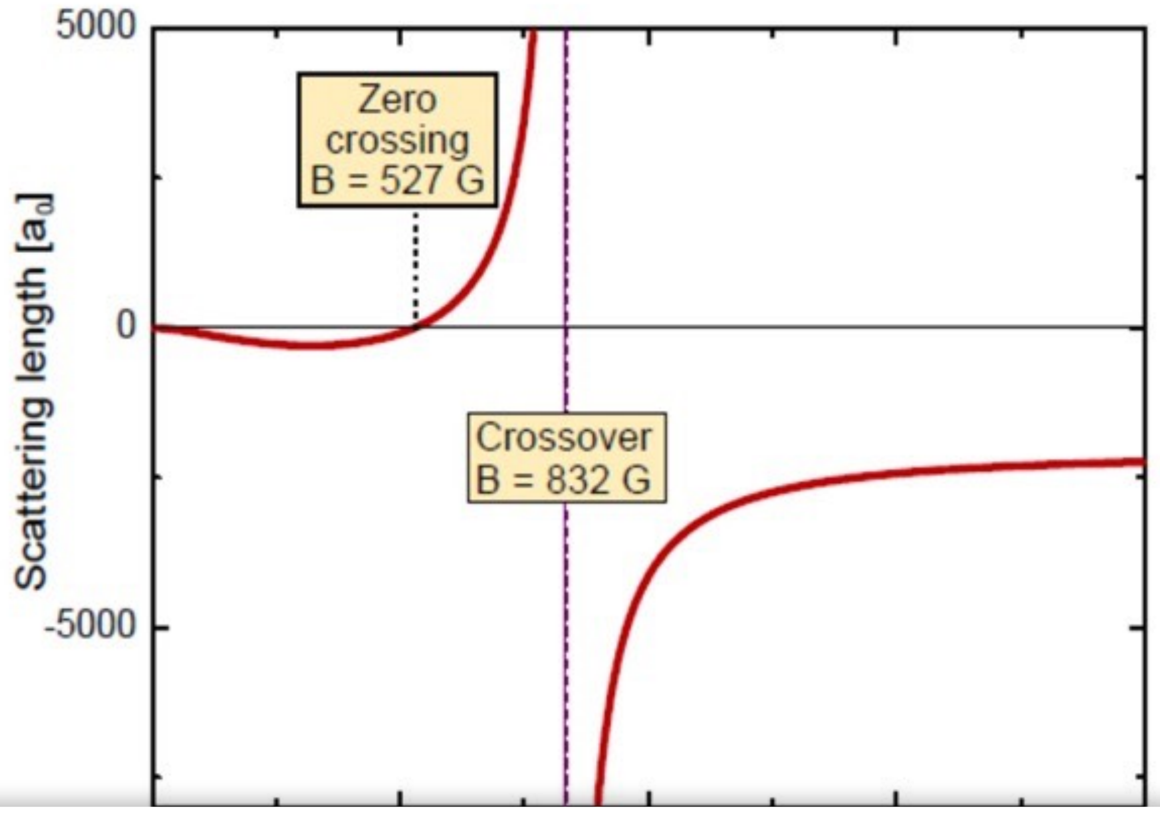


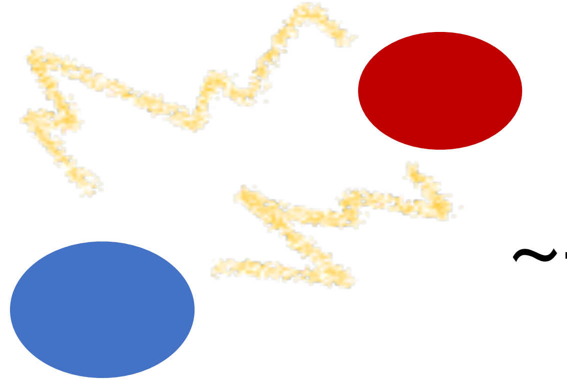
Feature	671 nm	532/1064 nm
Linewidth	77 kHz	1.4 MHz
Finesse	47 000	2 800
Mode waist	45 um	50 um



N ~ 10^5 at T ~ 30 nK

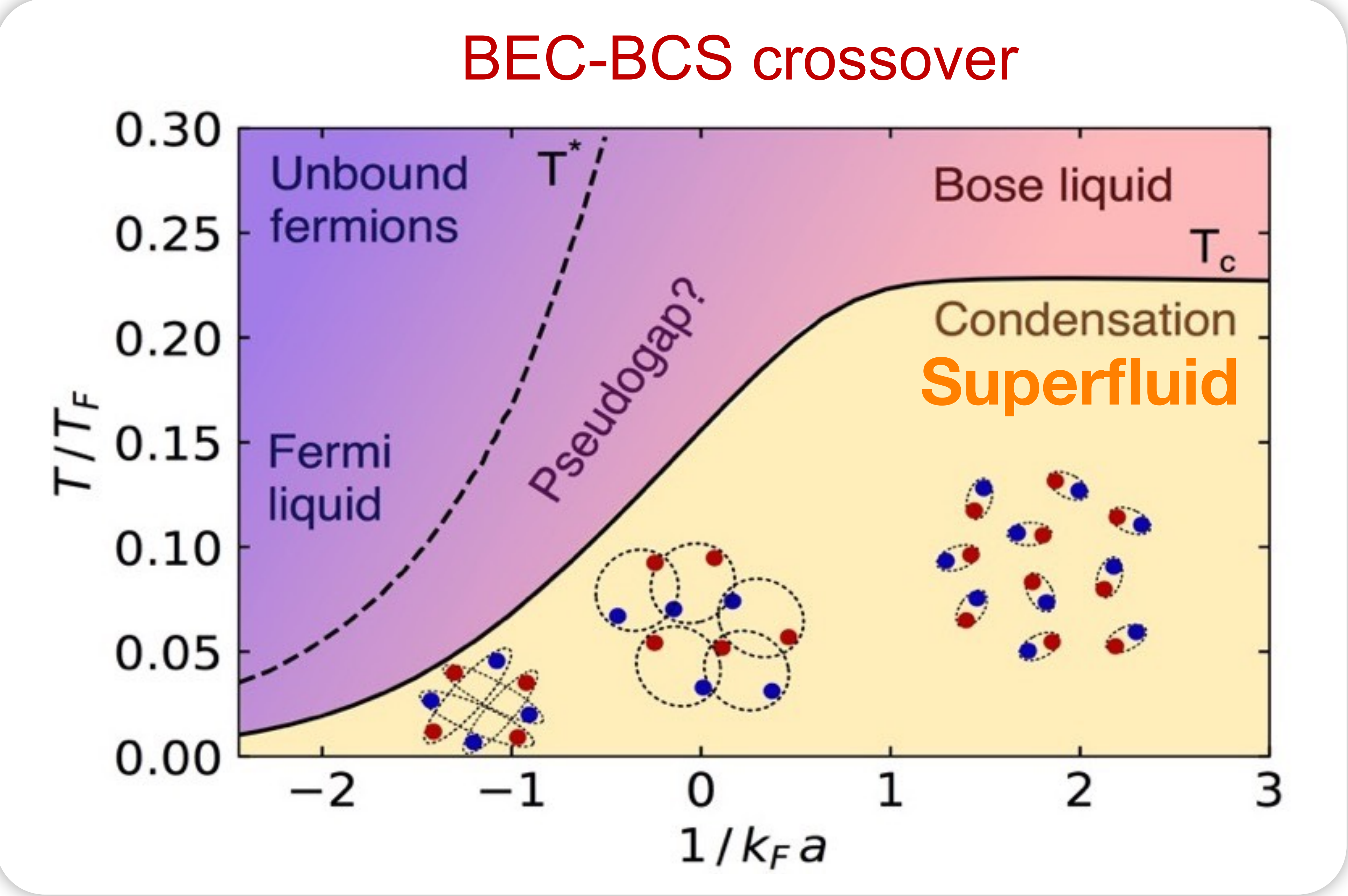






$$\sim \frac{4\,\pi\,\hbar^2}{m}\,a\,\delta($$

$a \longrightarrow$ scattering



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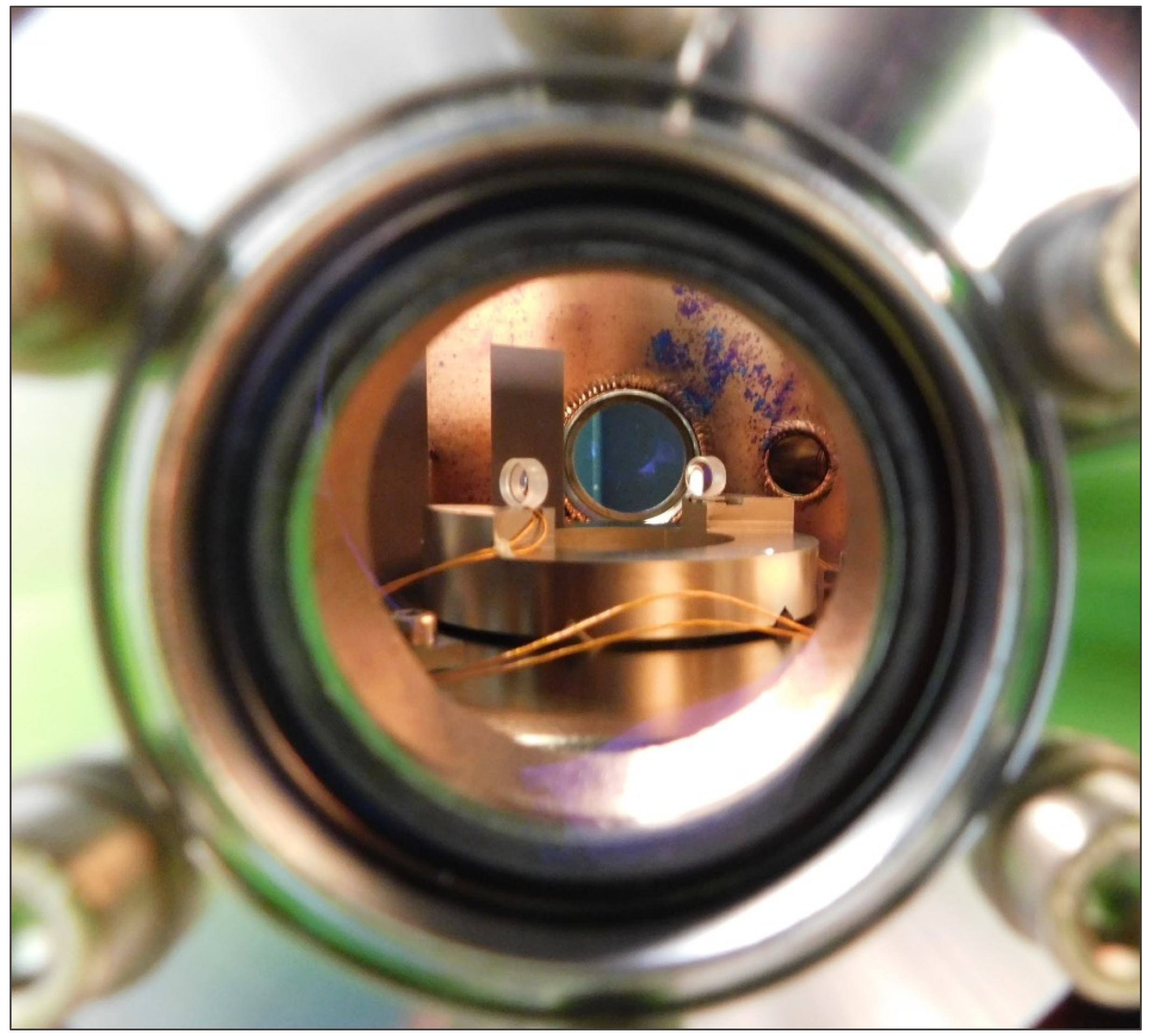

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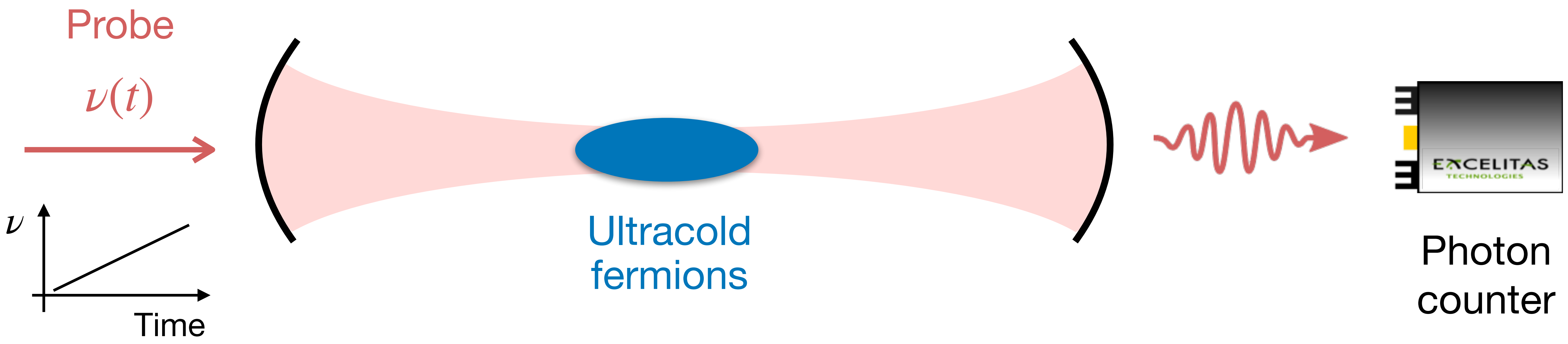
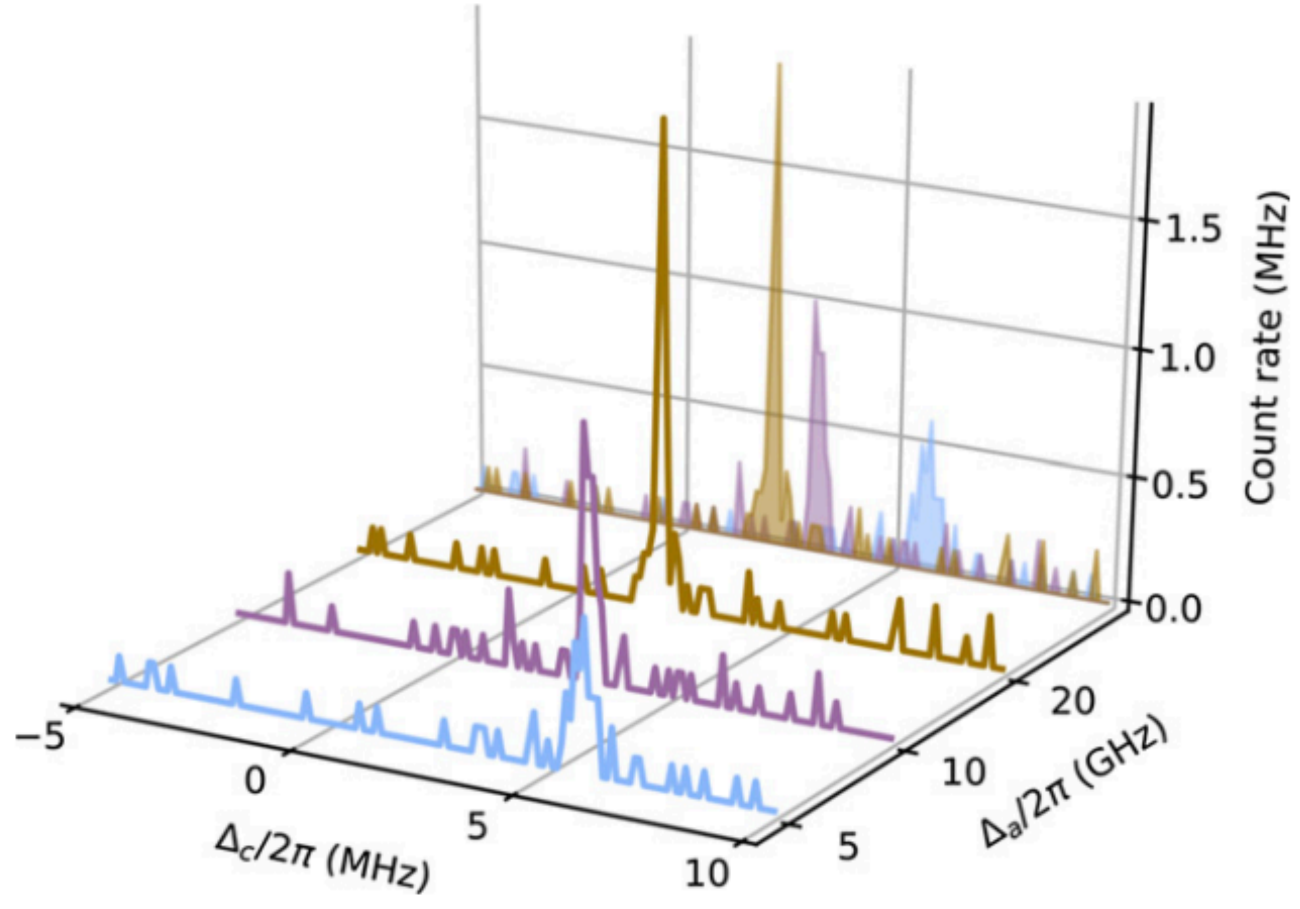
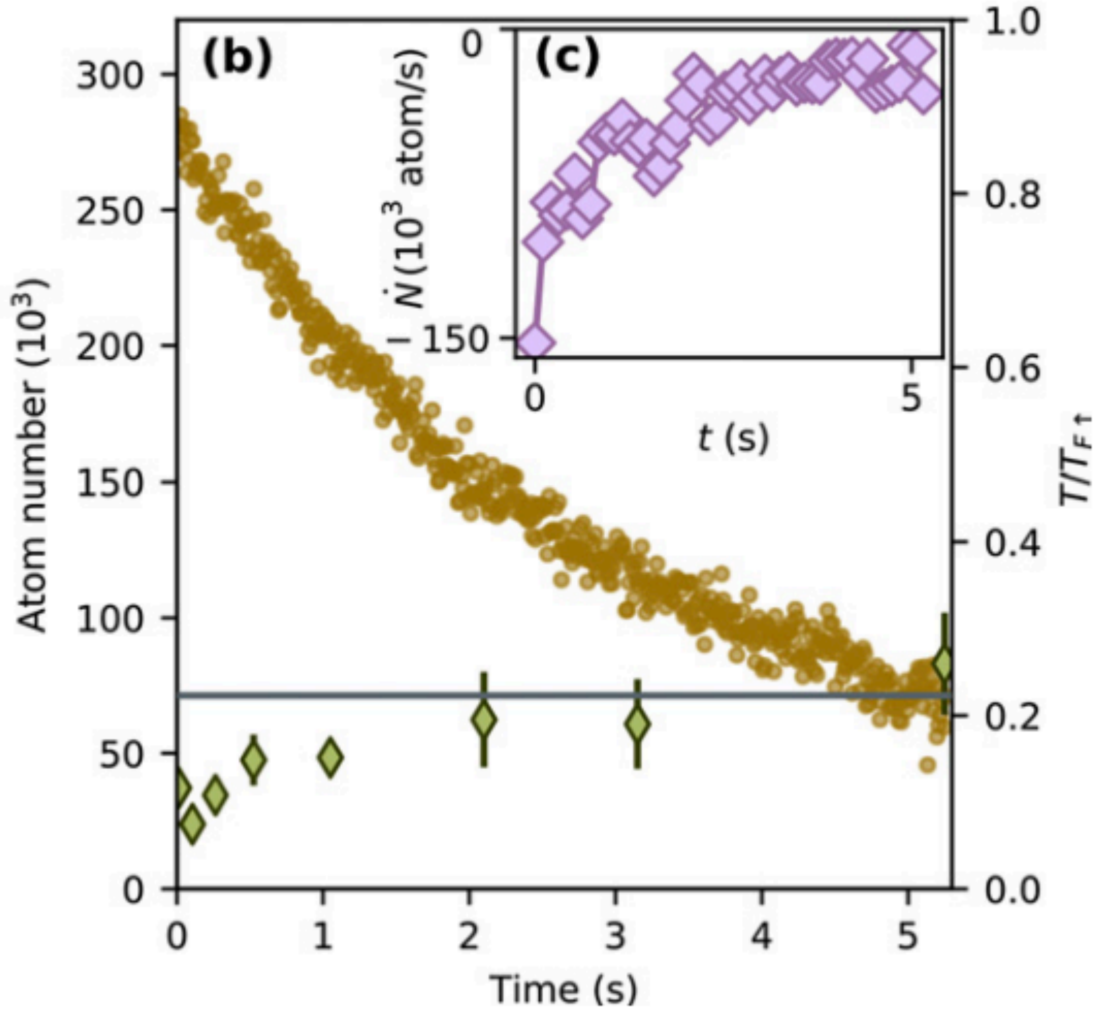


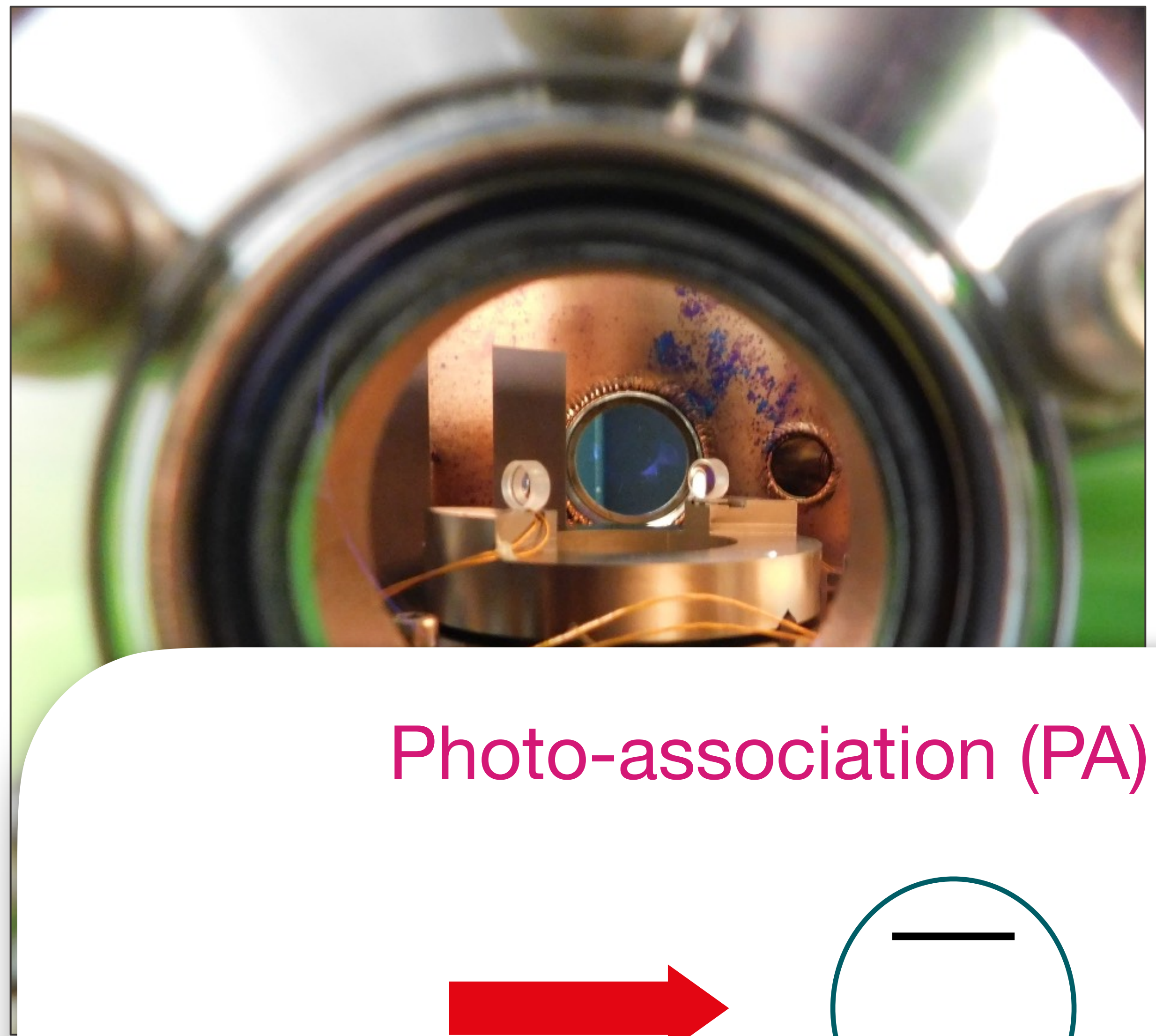
1 **Dispersive regime:** non-destructive measurements of the lifetime

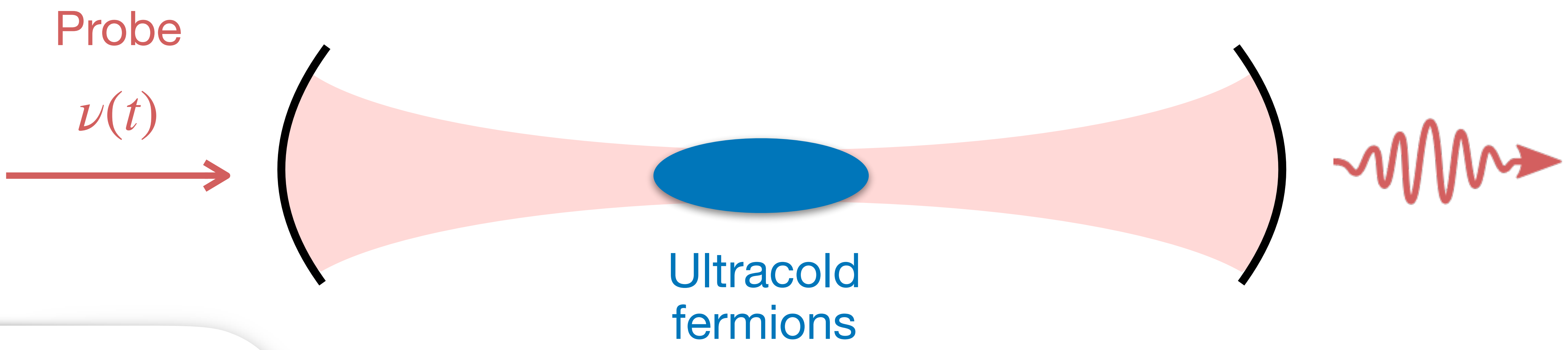




K. Roux, et al., *New Journal of Physics* 23.4 (2021): 043029.

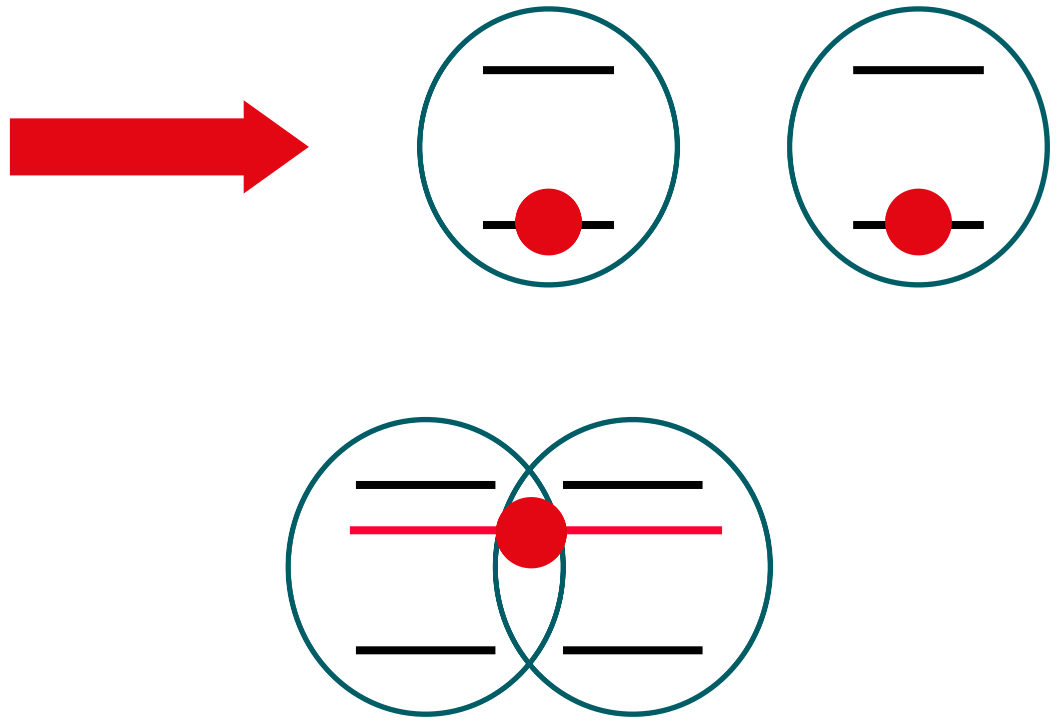
CAVITY TRANSMISSION SPECTROSCOPY ON ULTRACOLD FERMIONS





Photon counter

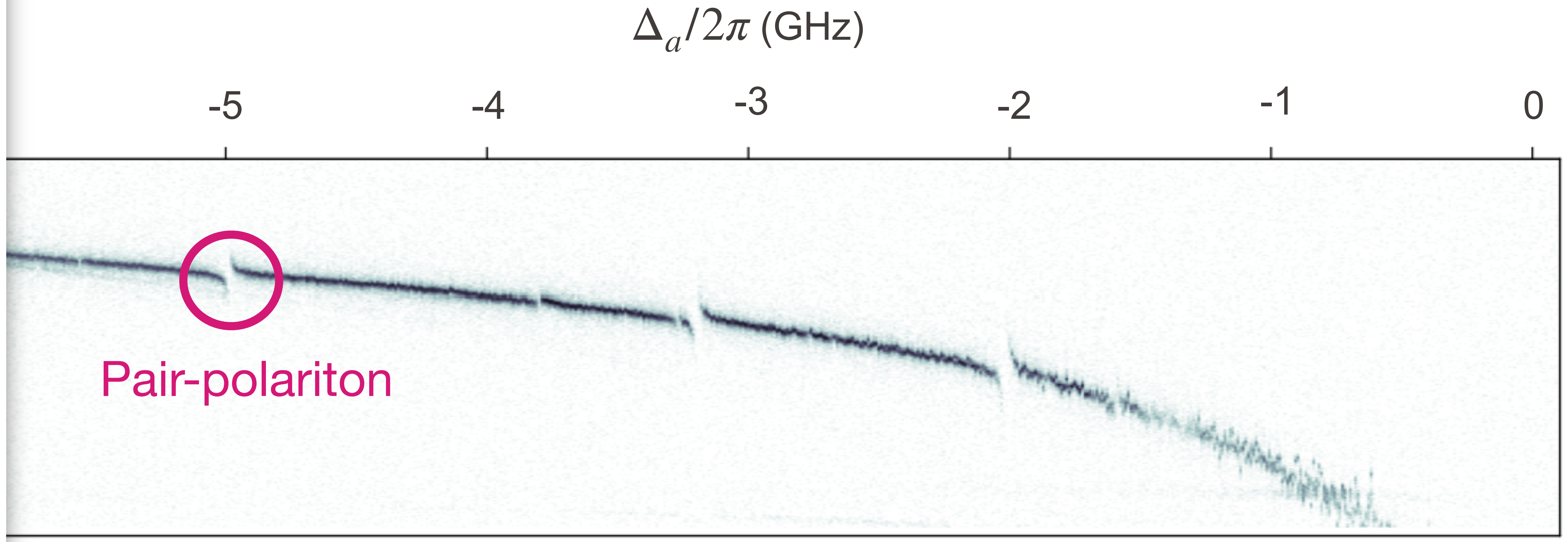
Photo-association (PA) transition



A photon is absorbed by a pair of atoms that are lifted to a **highly excited molecular state**

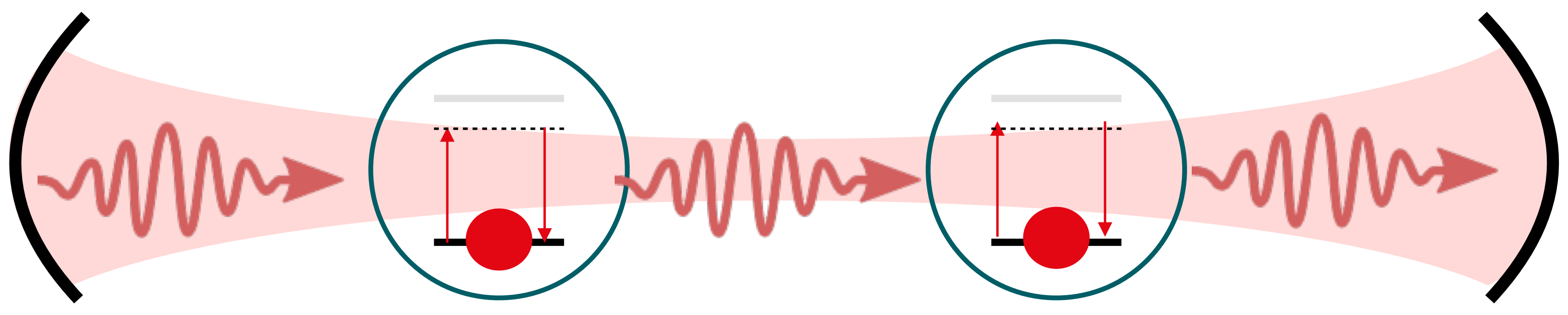
Review on PA spectroscopy:
K. M. Jones, et al., Rev. Mod. Phys. 78, 483–535 (2006).

ant regime: Strong light-matter coupling.



H. Konishi, et al., *Nature* 596, 509–513 (2021).

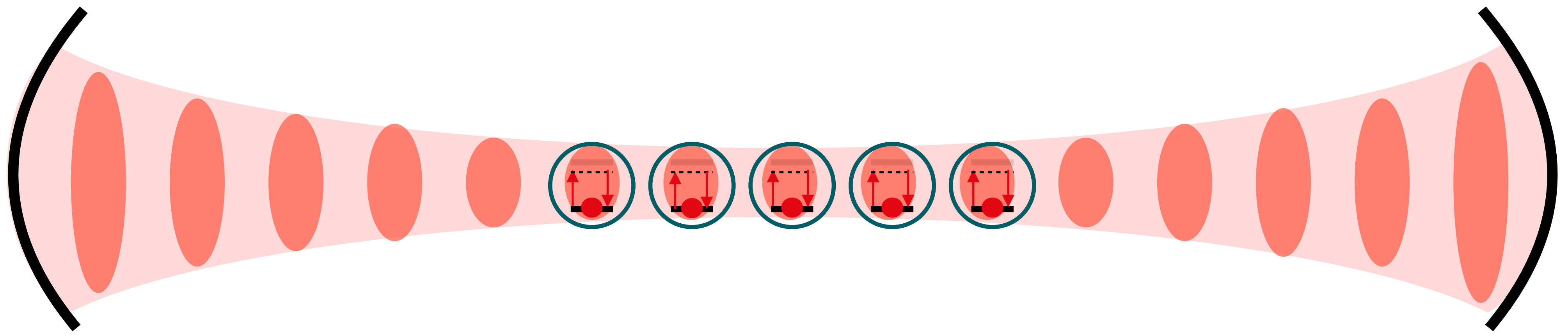
CAVITY MEDIATED LONG-RANGE INTERACTIONS



Two atoms in the cavity can **coherently exchange a photon**, that mediates an effective interaction between them. Such interaction between atoms:

- is a **long-range interaction**: any two atoms in the cavity volume can exchange cavity photons
- has a strength proportional to the **number of photons** in the cavity
- induces the **self-organization** phase transition

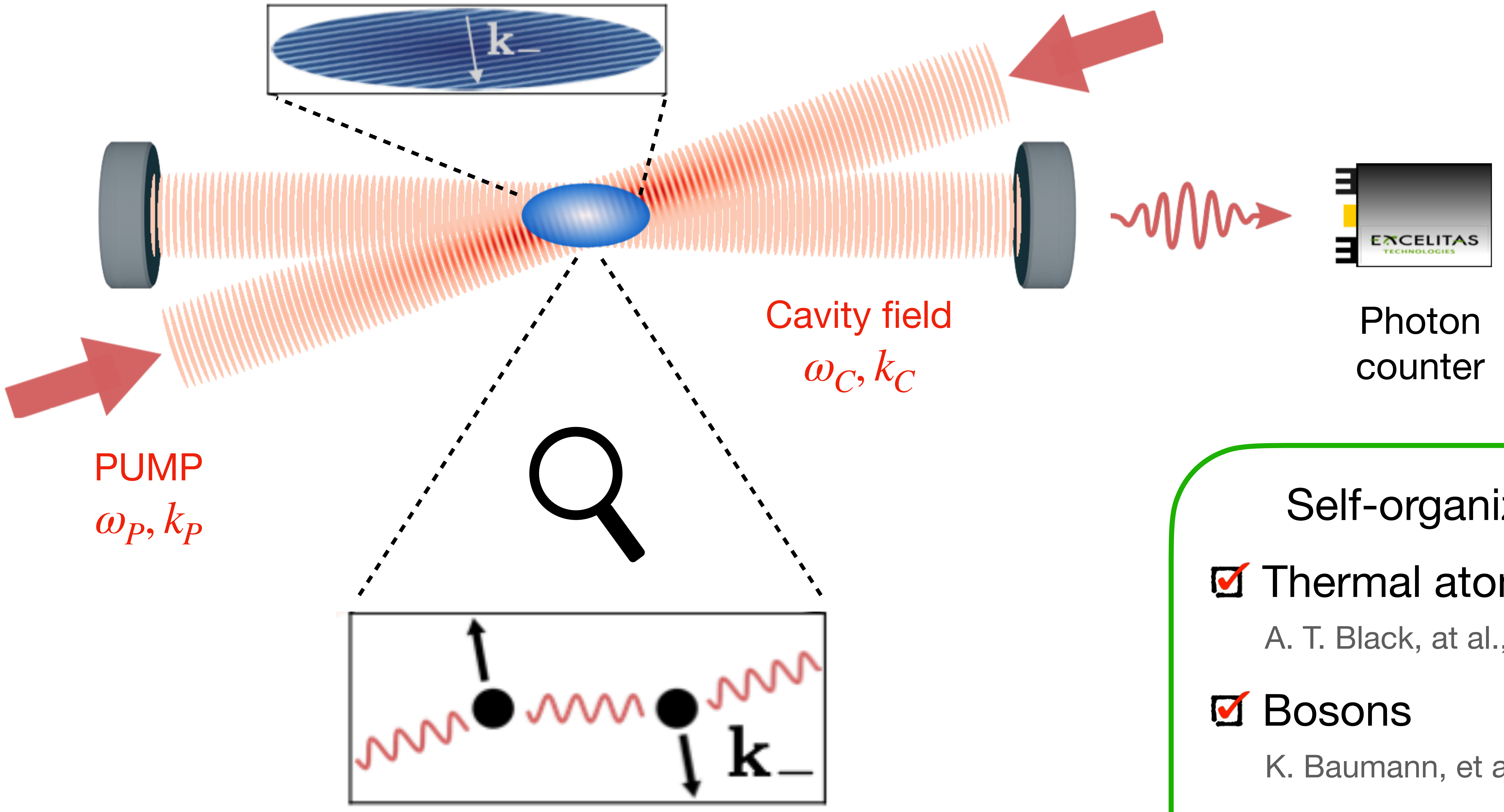
Self-organization phase transition

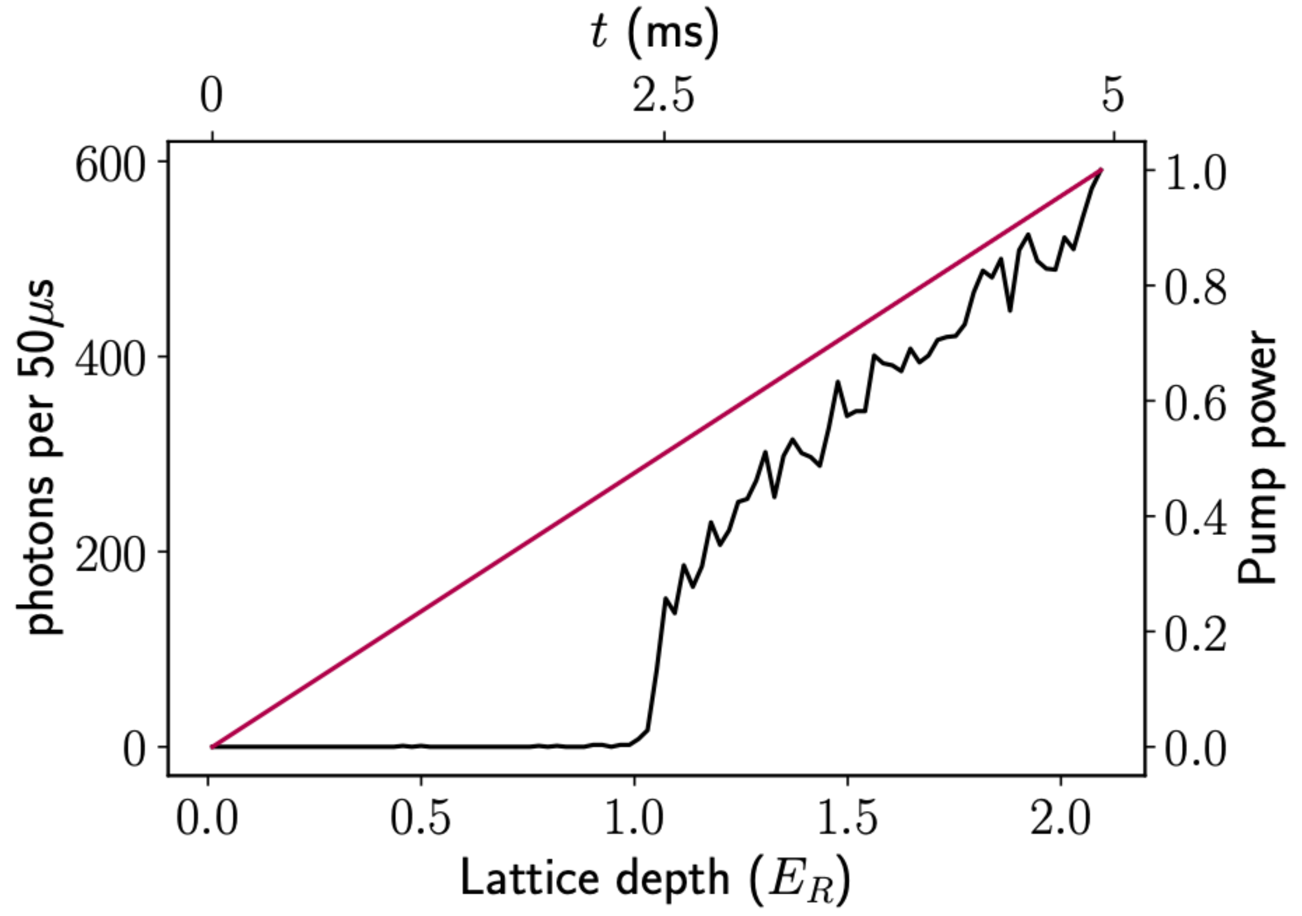


For red-detuned light with respect to the atomic transition:

- The light forms a standing wave inside the cavity
- The light exert an attractive potential to the atomic sample
- The atoms form a crystal, once driven by an external pump

SELF-ORGANIZATION PHASE TRANSITION



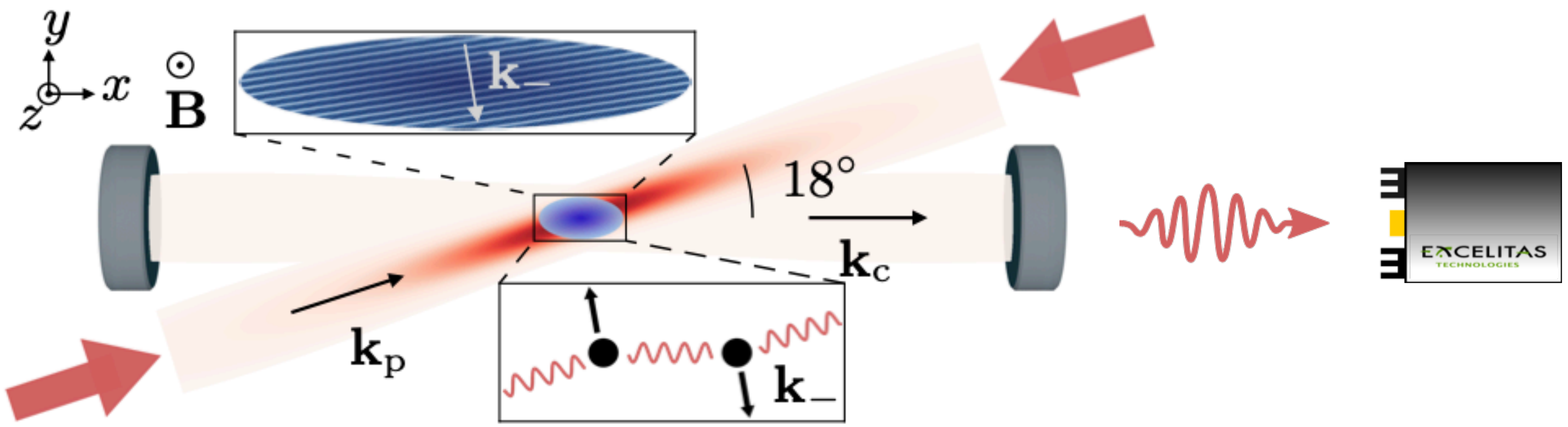


Self-organization = spontaneous density-wave order

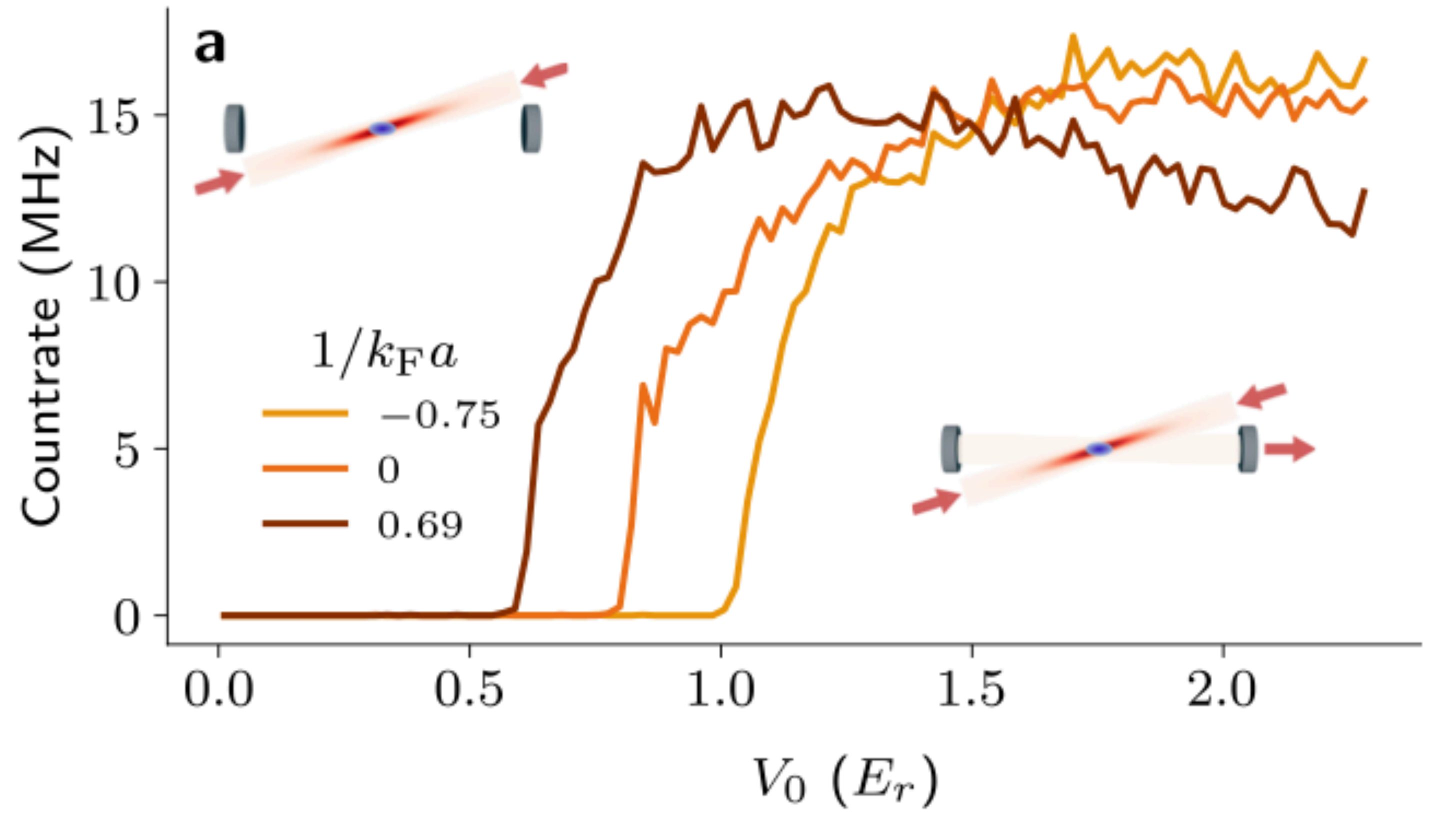
Self-organization phase transition:

- ☑ Thermal atoms
  A. T. Black, at al., Phys. Rev. Lett. 91, 203001 (2003).
- ☑ Bosons
  K. Baumann, et al., Nature 464, 1301 (2010).
- ☑ Non-interacting fermions
  X. Zhang, et al., Science 373, 1359 (2021).
- ☑ Strongly-interacting fermions
  V. Helson et al., arXiv:2212.04402

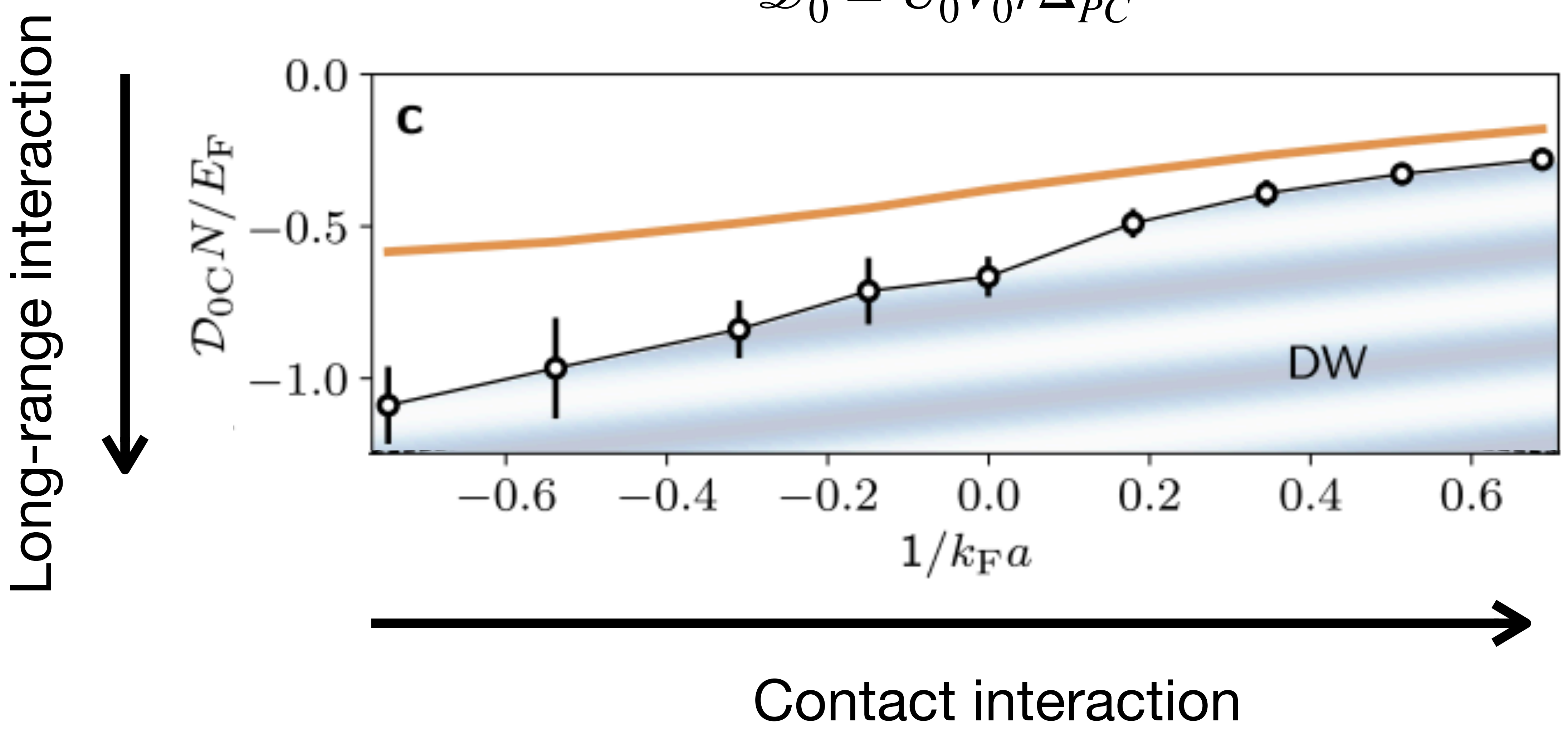
SELF-ORGANIZATION OF ULTRACOLD FERMIONS

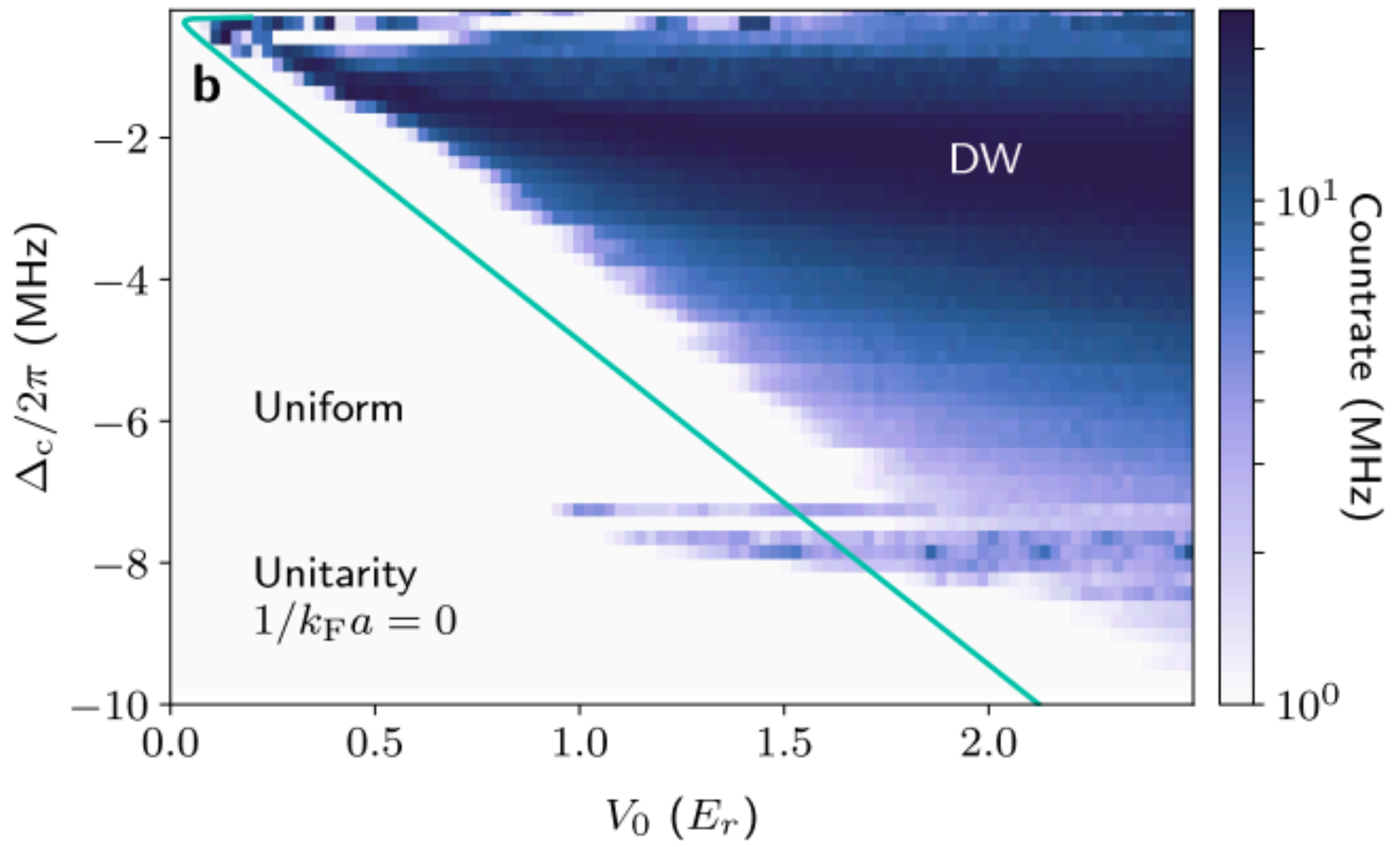


$\Delta_{PC} = -2 \times 2\pi$ MHz



$\mathscr{D}_0 = U_0 V_0 / \Delta_{PC}$

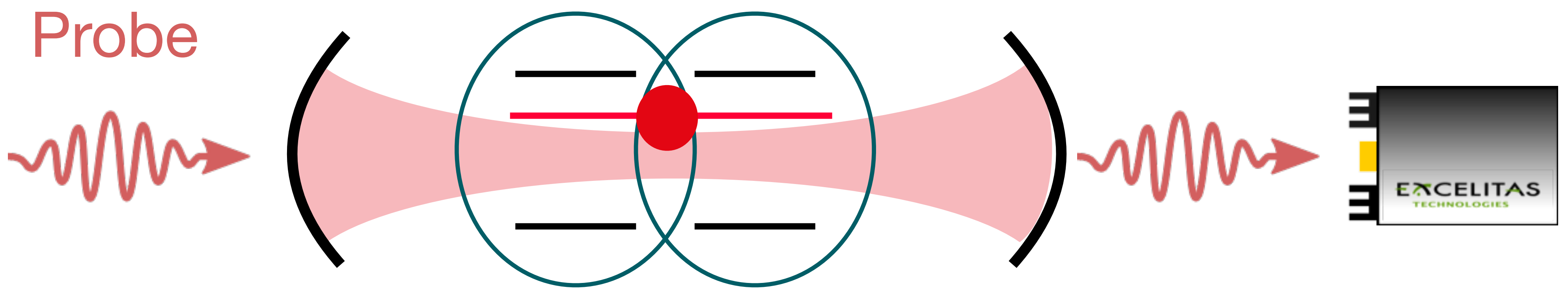




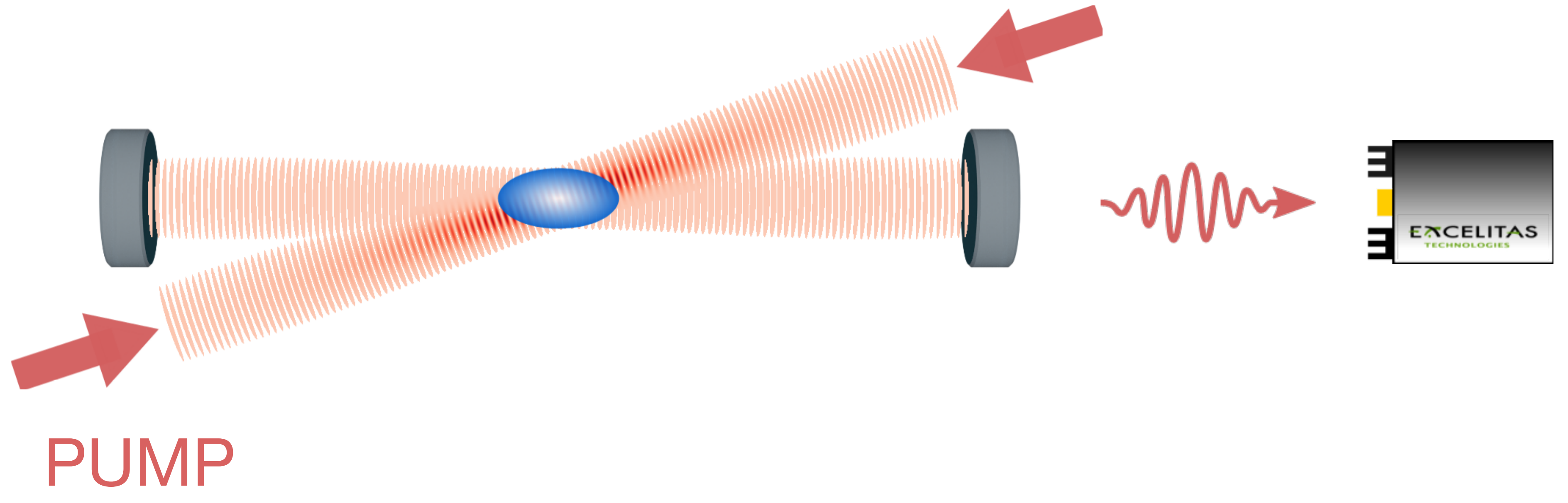
V. Helson et al., arXiv:2212.04402

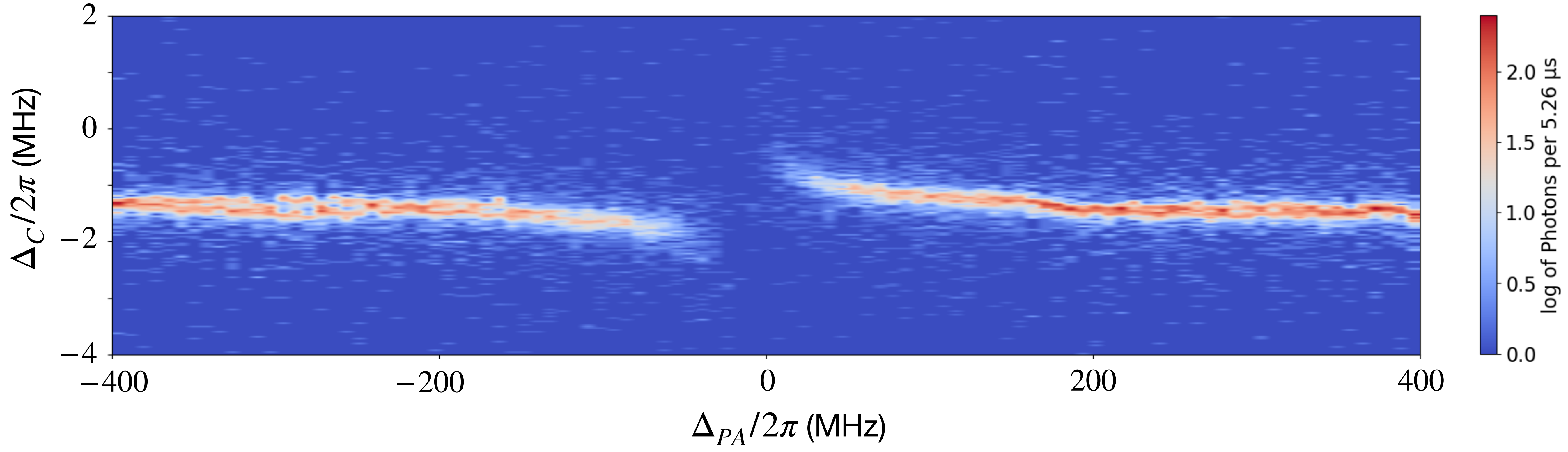
SELF-ORGANIZATION CLOSE TO A PA TRANSITION

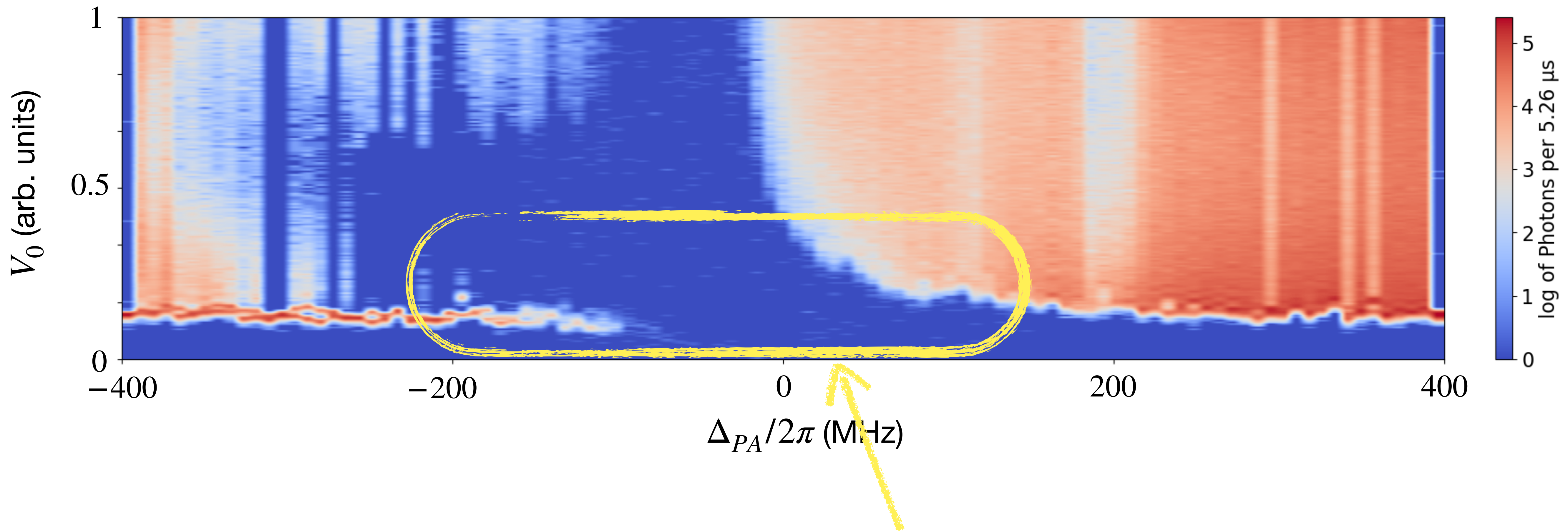
Cavity spectroscopy



Self-organization trace

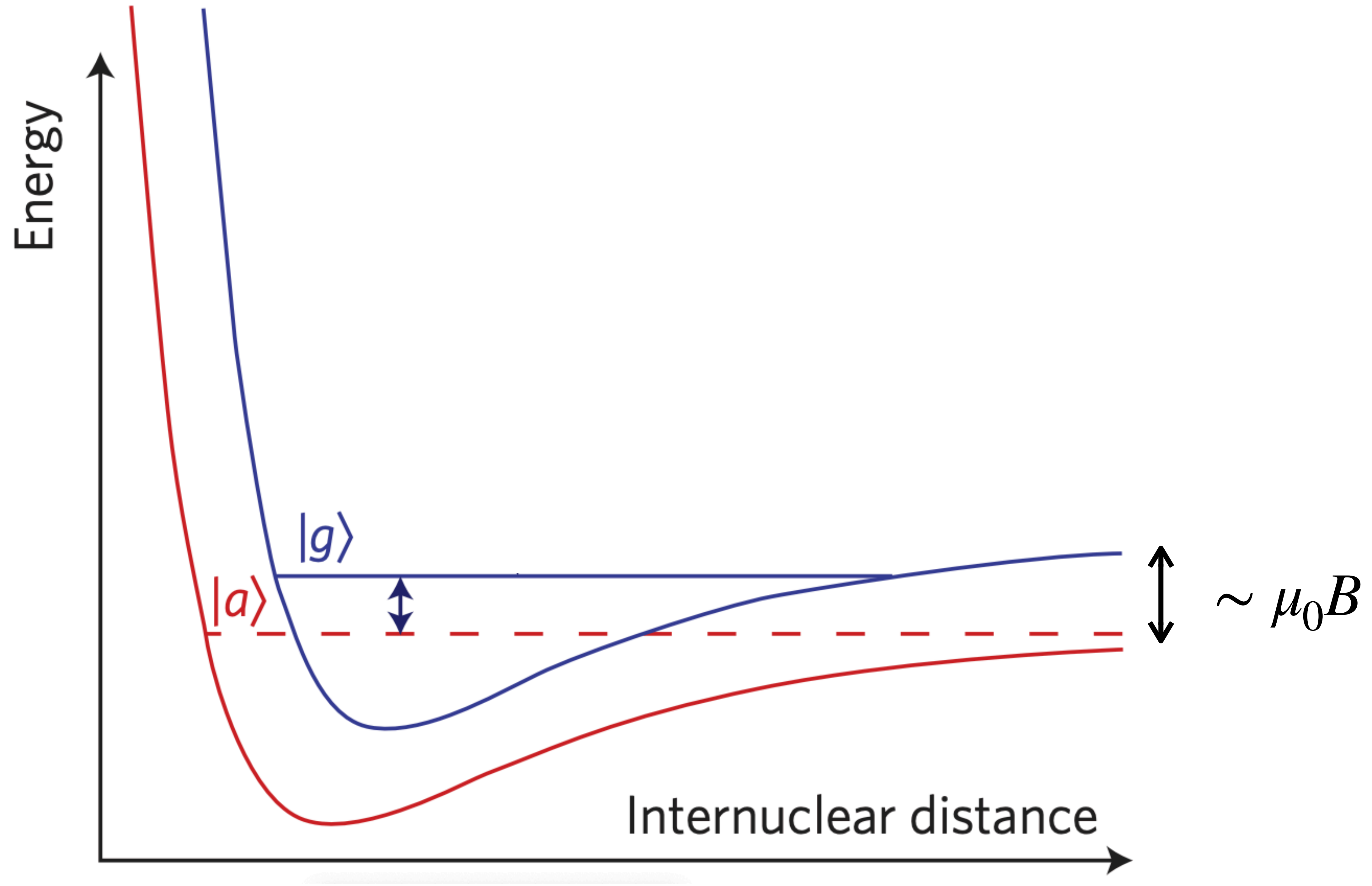






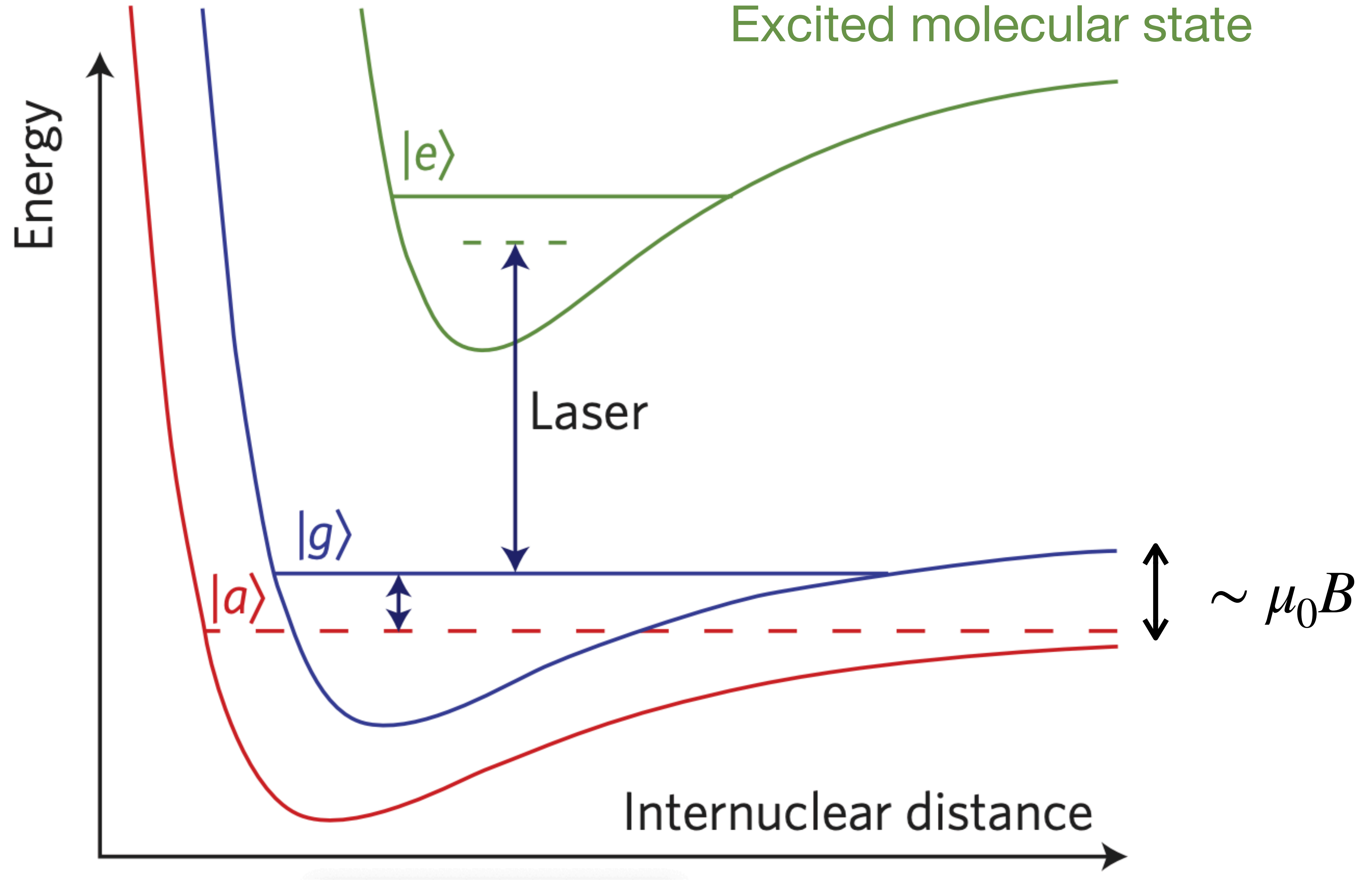
The critical pump strength for self organization changes in proximity of the PA line

PA AS OPTICAL FESHBACH RESONANCES



D. Bauer, et al., *Nature Physics* 5.5 (2009): 339-342.

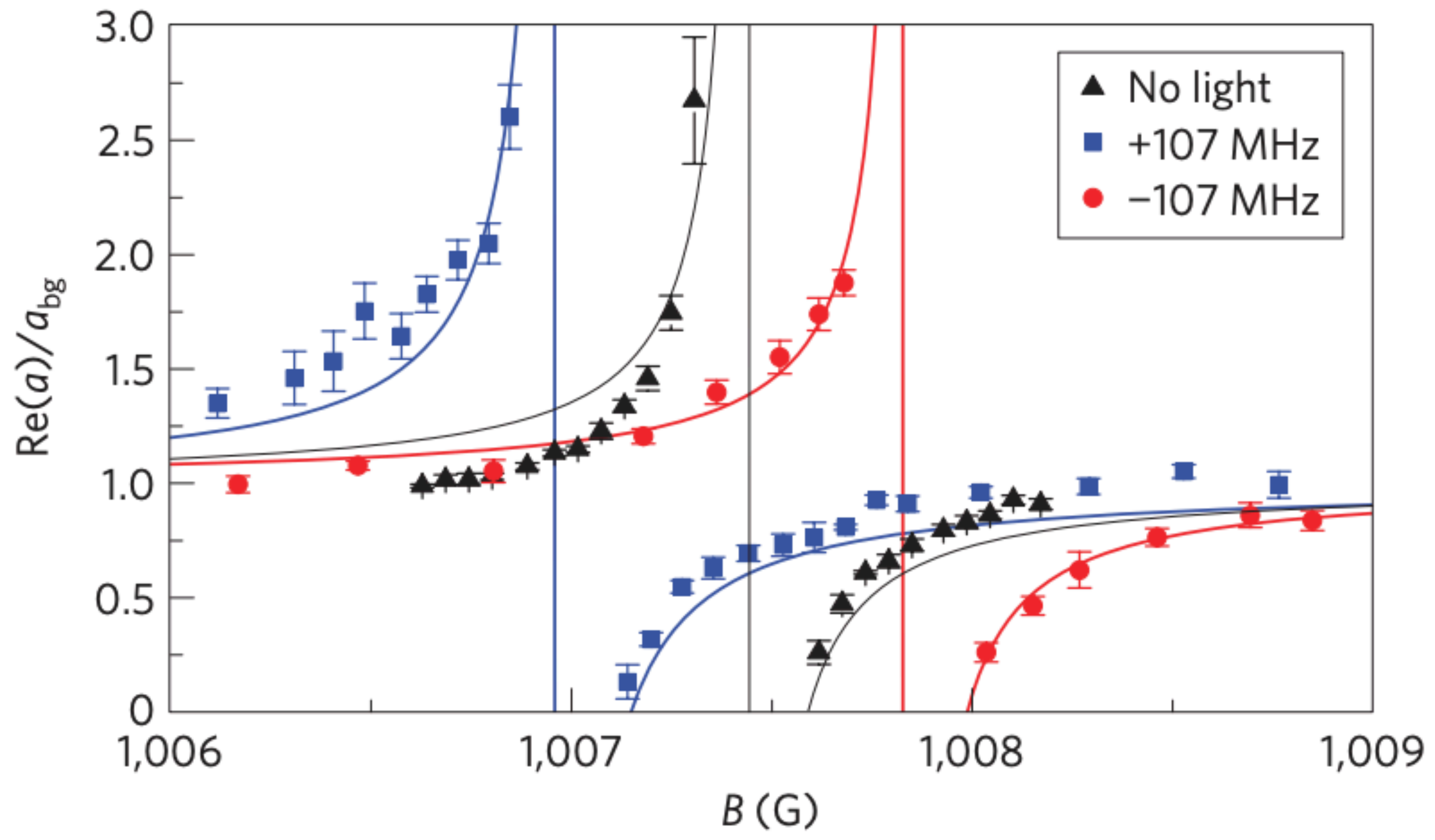
PA AS OPTICAL FESHBACH RESONANCES



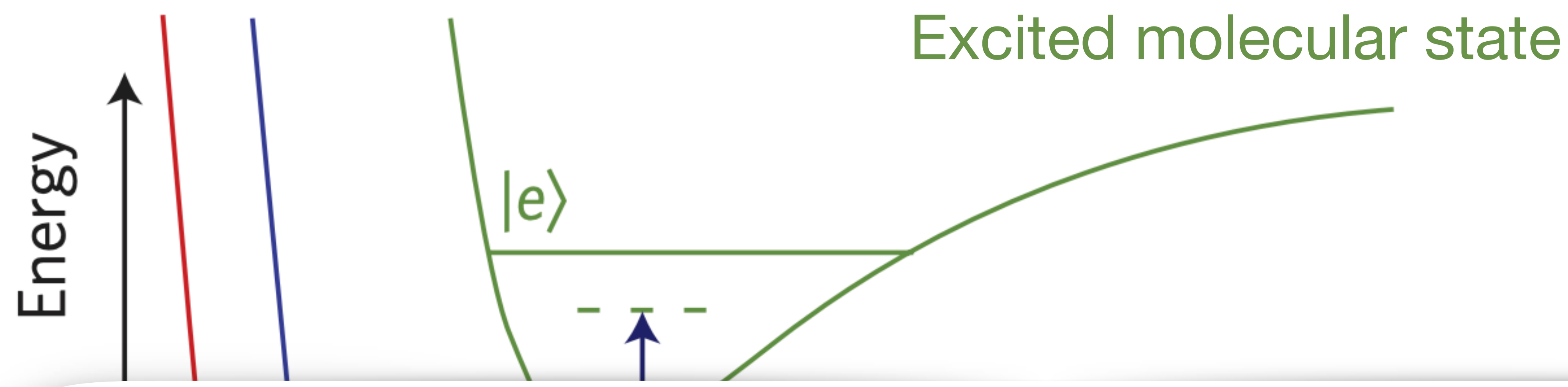
D. Bauer, et al., *Nature Physics* 5.5 (2009): 339-342.

The laser that couples $|g>$ to $|e>$ shifts the position of $|g>$ creating a **dressed state** with an energy shift $\sim \Omega^2 \sim I$

Tuning the intensity of the laser we can tune the relative $|g> - |a>$ energy shift, changing the effective location in B field of the magnetic Feshbach resonance



PA AS OPTICAL FESHBACH RESONANCES



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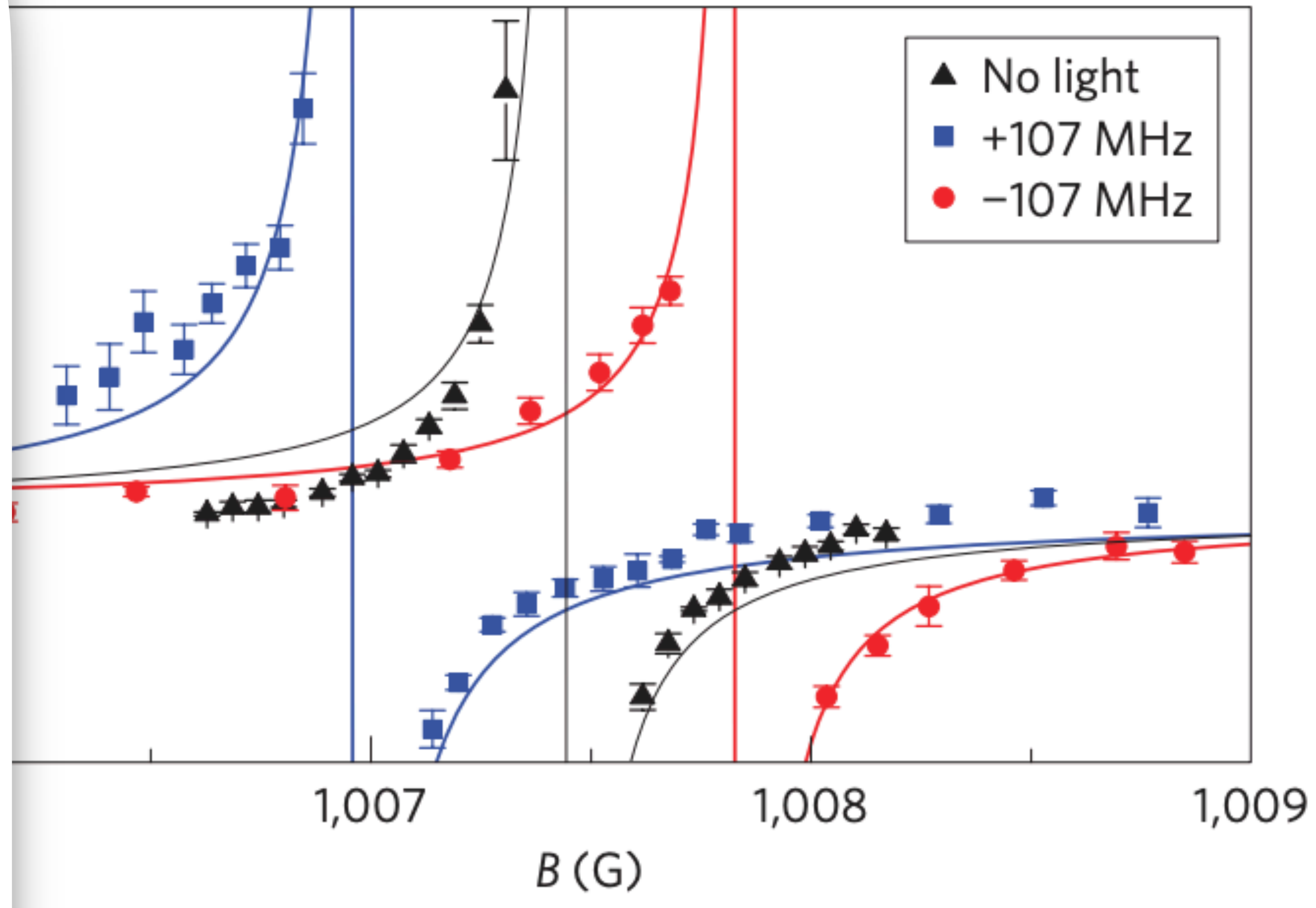
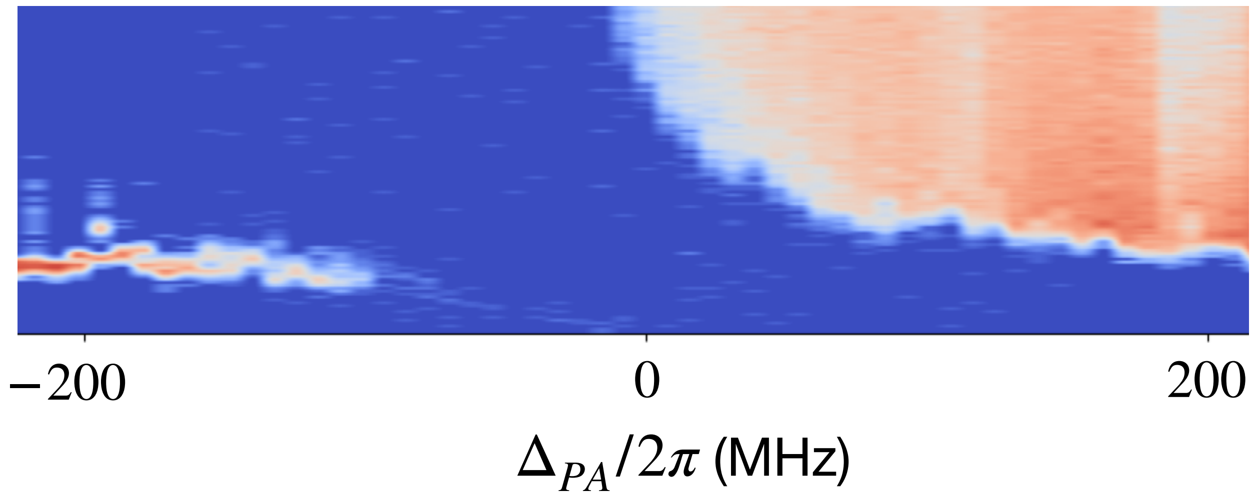
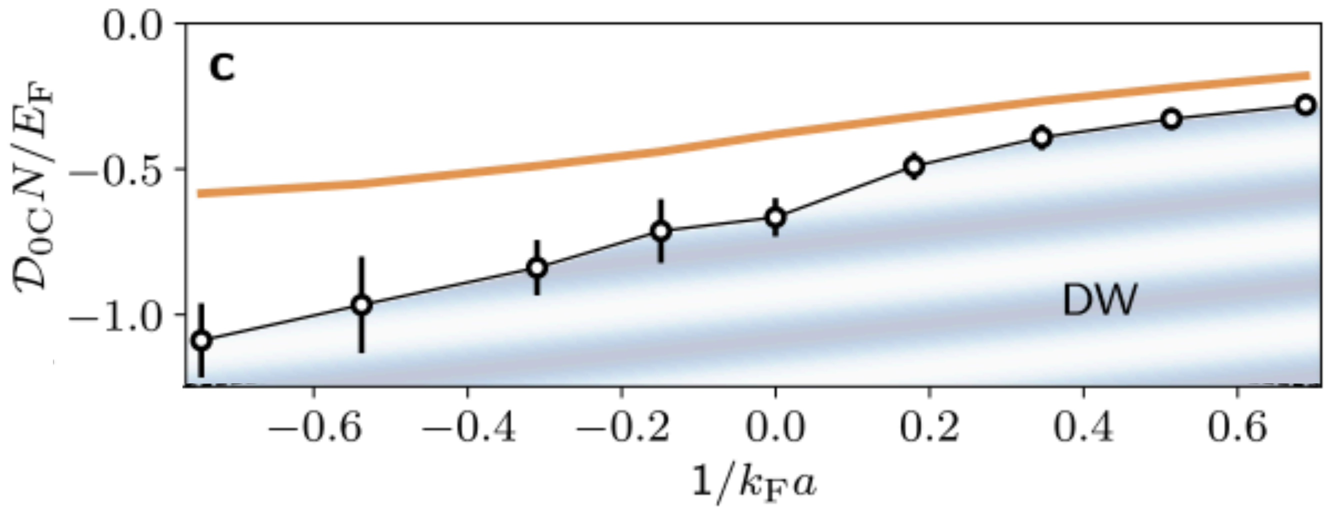
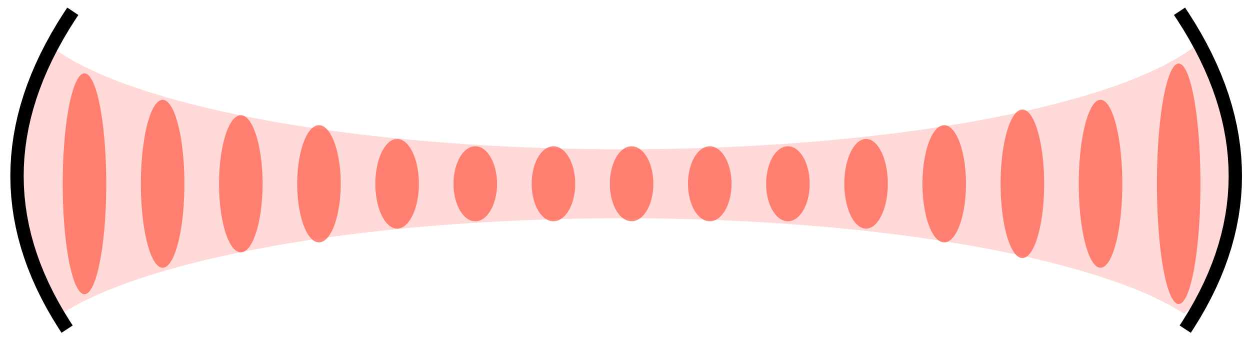


Self-organizzation close to PA: local modulation of the scattering length



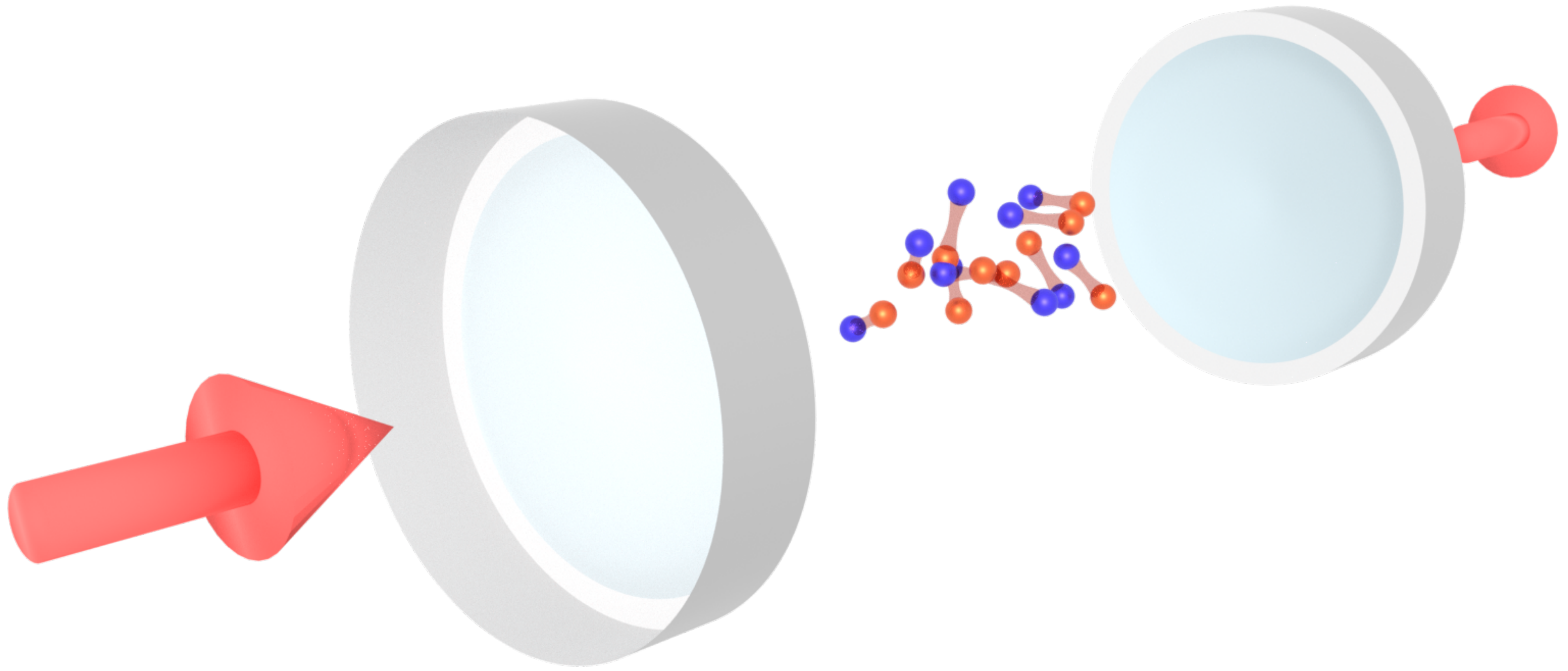




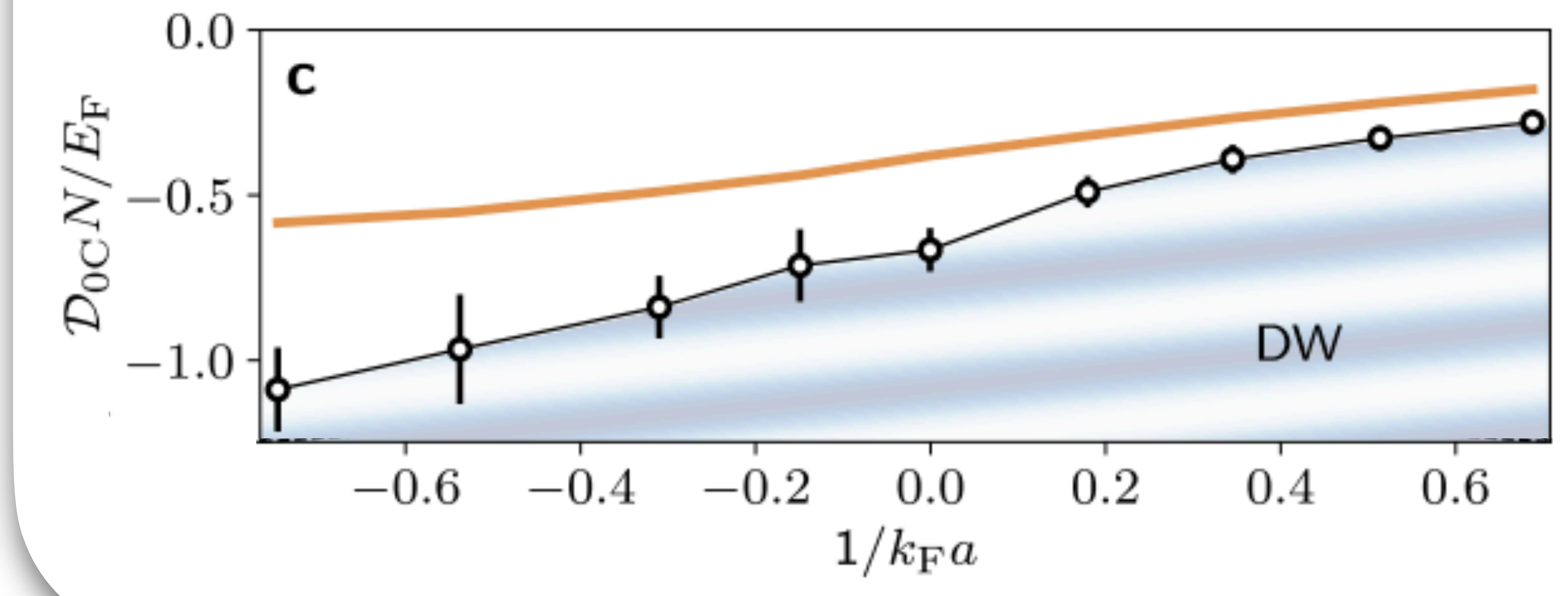
The modulated intensity inside the cavity induces a modulation of the scattering length, and thus **a modulation of pairing**

CONCLUSIONS

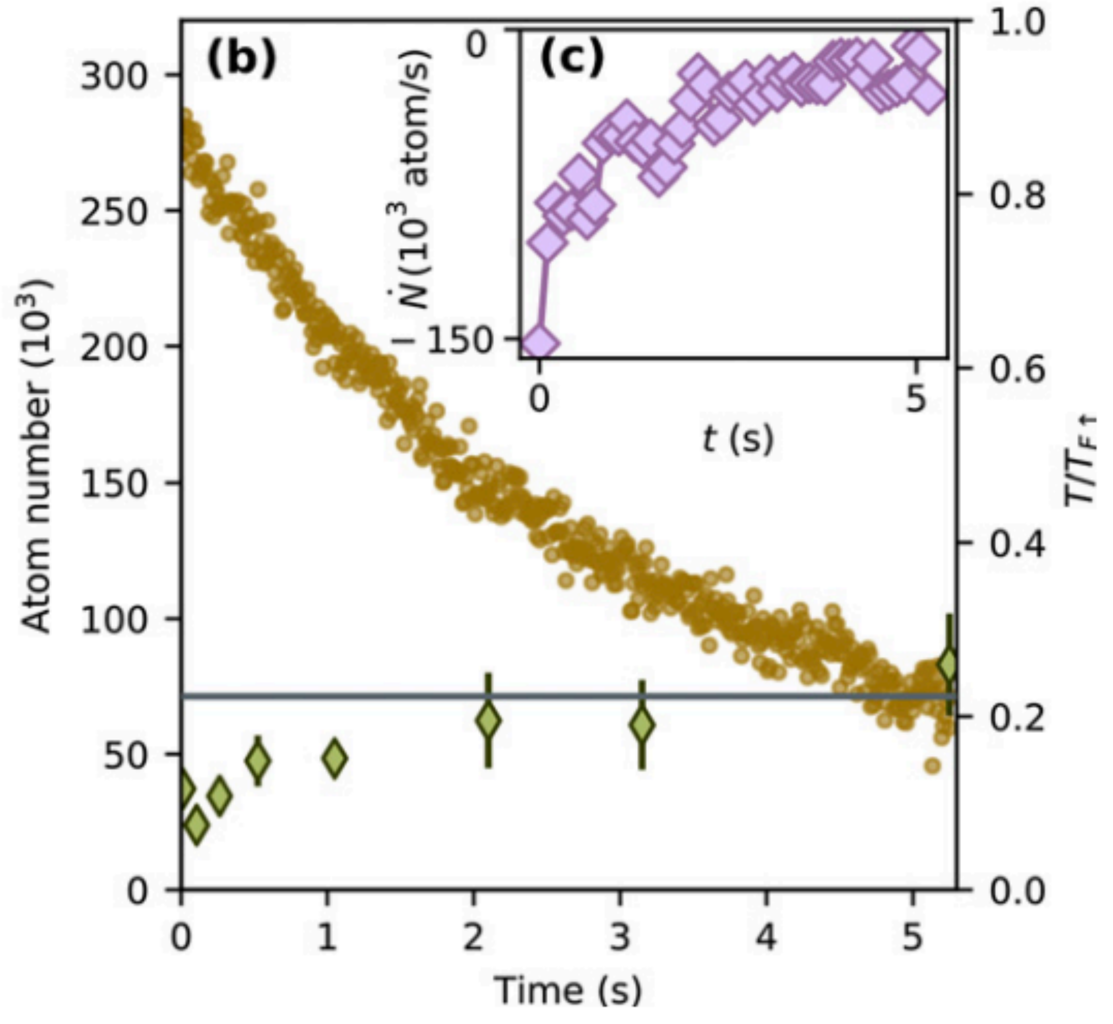
Ultracold atoms coupled to optical cavities



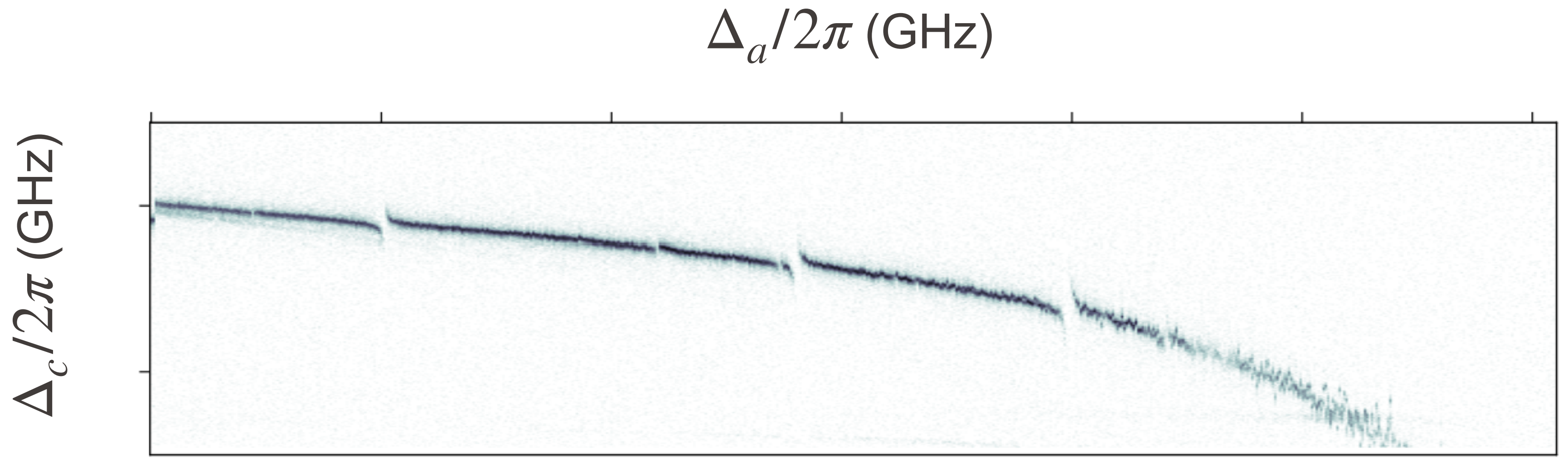
Long-range interactions



Weakly-destructive probe



Strong light-matter coupling



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THANK YOU FOR YOUR ATTENTION!