Can we detect deep axisymmetric toroidal magnetic fields in stars?

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Importance of angular momentum transport in stars

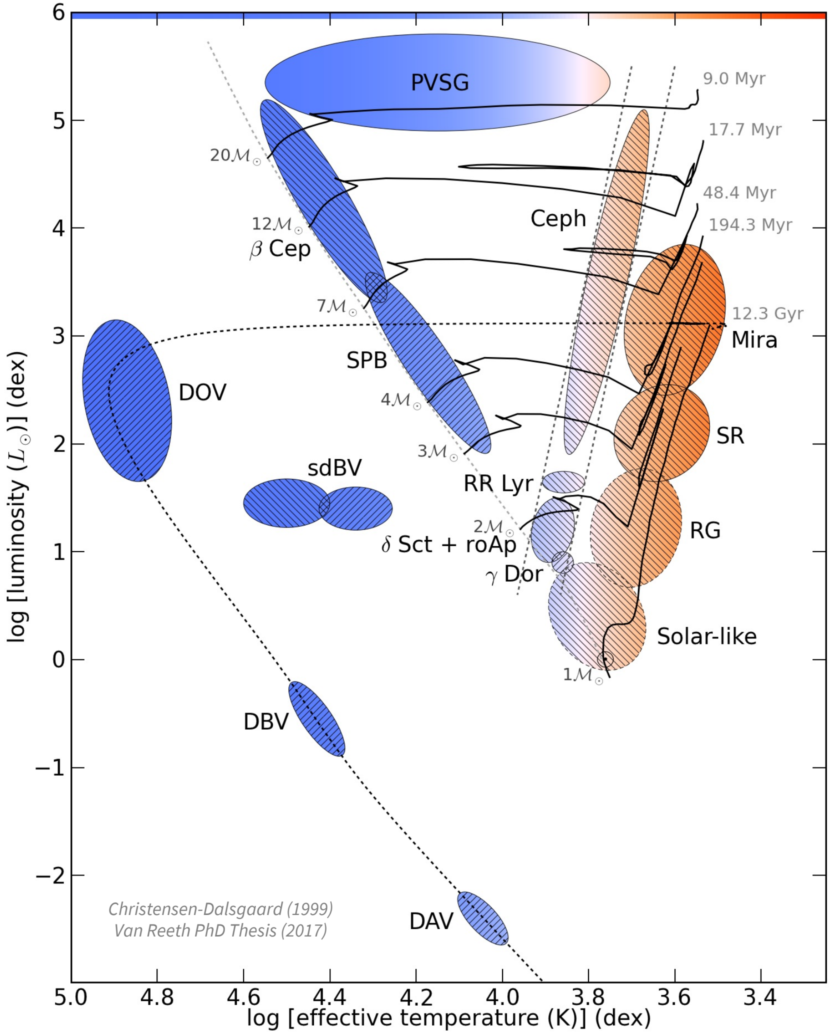
- ❖ Stars: **fundamental units** of the visible Universe
  - ➢ Stellar structure and evolution: **cornerstone of Astrophysics**

- **Goal:**
  - ▪ Understand stars interiors, dynamics, and evolution

- **Key element:**
  - ▪ Transport of angular momentum

- **Why?**
  - ▪ Angular momentum transport impacts:
    - ✓ rotation
    - ✓ chemical mixing
    - ✓ evolution
    - ✓ interactions with their environment
    - ✓ magnetism

Angular momentum transport shapes the evolution and the dynamics of stars



Asteroseismic HR Diagram

Magnetic fields in radiative zones of early-type stars

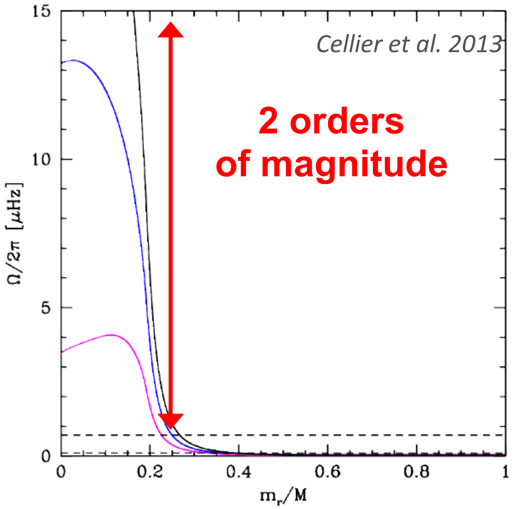
- Current angular momentum transport models unable to reproduce asteroseismic observations *(Aerts et al. 2019)*

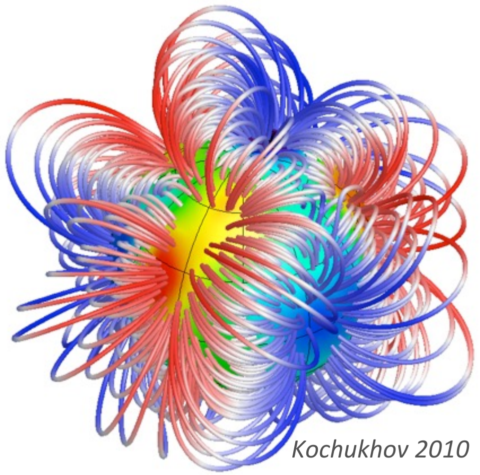
- One of the best candidates to explain this discrepancy: **Magnetic fields** in radiative zones

- 2 types of detected magnetic fields at stellar surfaces using spectropolarimety:

  - Large-scale stable fields with high amplitude
    - ✓ *Origin:* fossil *(Braithwaite & Spruit 2004, Duez & Mathis 2010, Shultz et al. 2019)*

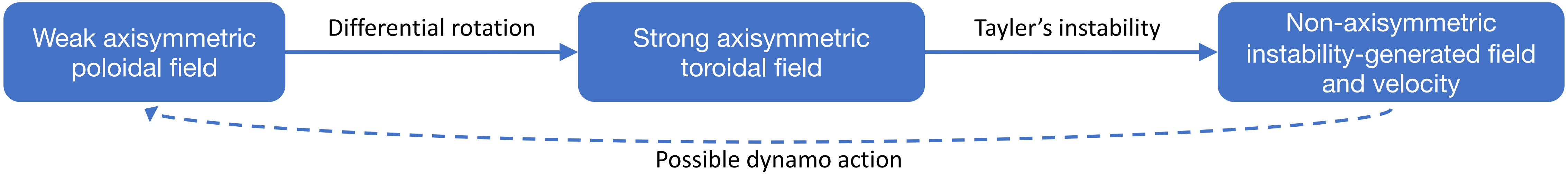
  - Small-scale fields with low amplitude
    - ✓ *Origin:* dynamo action in the thin sub-surface convective layer *(Cantiello & Braithwaite 2019)* / failed relaxation *(Braithwaite & Cantiello 2013)* / resulting from non-axisymmetric instabilities *(Aurière et al. 2007)*



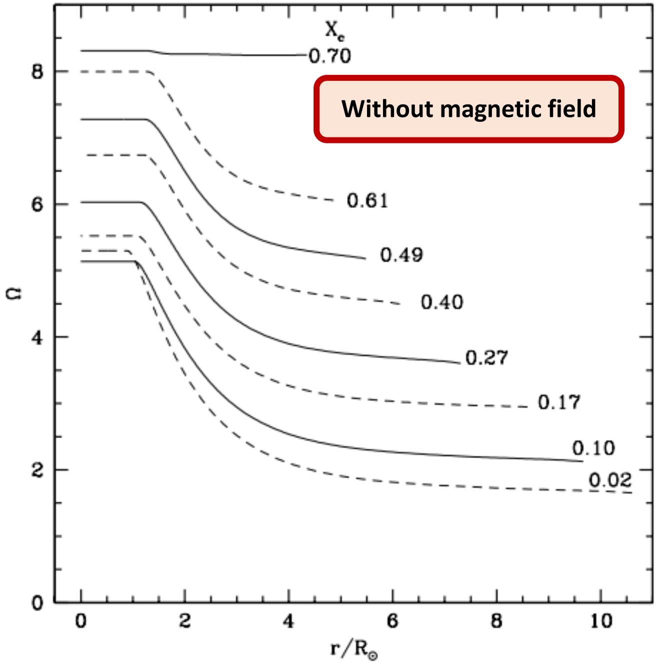


Strong axisymmetric toroidal magnetic field

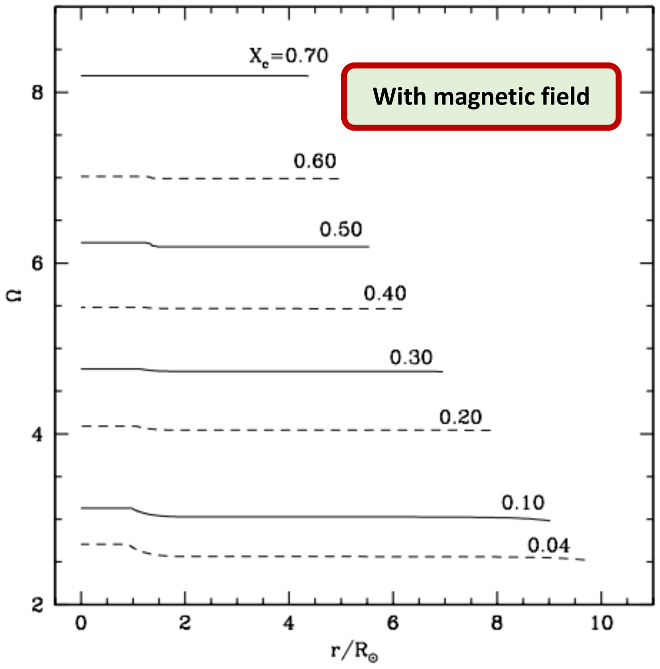
(Pitts & Tayler 1985, Spruit 2002, Zahn et al. 2007, Fuller et al. 2019, Petitdemange et al. submitted)



- **Torque exerted by induced Maxwell stresses: very efficient transport of angular momentum**



Enforce the solid-body rotation of massive MS stars



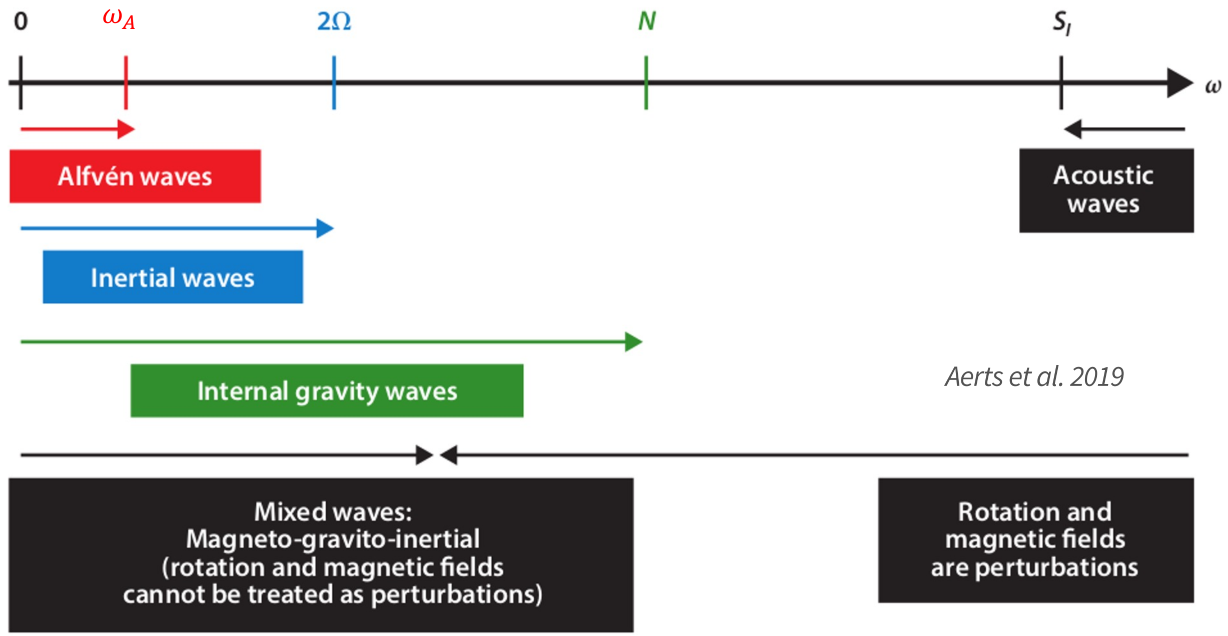
Maeder & Meynet 2004

Is it possible to detect this magnetic field?

This magnetic field does not emerge at the stellar surface
➥ **spectropolarimetry is blind**

Only window: magneto-asteroseismology
➥ look for characteristic magnetic signatures in stellar oscillations

g-mode pulsators and magnetic field



Aerts et al. 2019

- Magneto- Gravito-inertial (MGI) modes driven by:
  - ✓ buoyancy: chemical and thermal stratifications (N)
  - ✓ rotation: Coriolis and centrifugal accelerations (2Ω)
  - ✓ magnetic field: Lorentz force (ω_A)

- Acoustic modes driven by:
  - ✓ Compressibility: pressure force (S_ℓ)

Asteroseismology: only way to probe the internal **structural, chemical, rotational and magnetic properties**

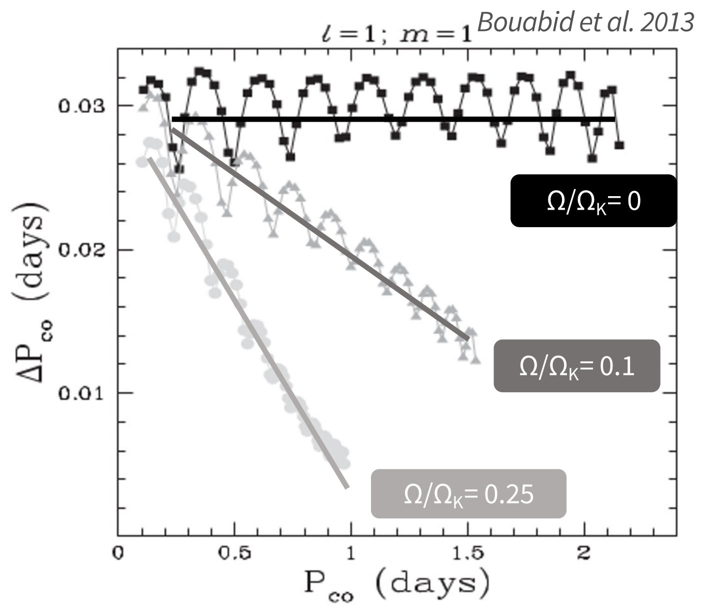
Goal: study the signature of **axisymmetric toroidal magnetic field**

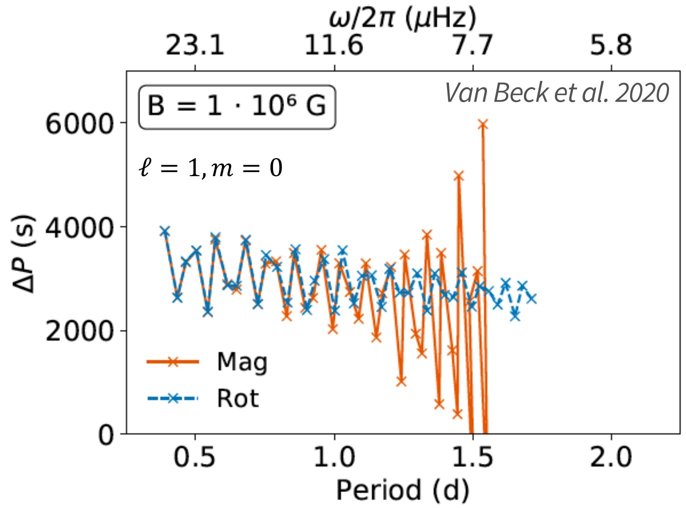
Period spacing pattern: a window into stars' interiors

$\Delta P = P_{n+1} - P_n$: period spacing

- No rotation → Constant
- Rotation → Slope
  - Measurement of the slope ⇒ measure of Ω *(Bouabid et al. 2013, Van Reeth et al. 2016, Ouazzani et al. 2017)*
- Chemical composition *(Degroote et al. 2010, Pedersen et al. 2021)* / Differential rotation / Coupling between inertial modes in the convective core and g modes *(Ouazzani et al. 2020, Saio et al. 2021)* → Dips
- Mixed magnetic field (poloidal + toroidal) *(Prat et al. 2019,2020, Van beck et al. 2020)* → Sawtooth pattern
  - **Perturbative approach**

Derive a non-perturbative formalism to study the impact of strong general axisymmetric toroidal magnetic field





Traditional Approximation of Rotation (TAR)

- Goal: Compute the pulsations of MGI modes
- Issue: Difficulty of performing flexible and intensive seismic modelling with 2D oscillation codes *(e.g. Reese et al. 2006)*
- Possible solution: The Traditional Approximation of Rotation (TAR) *(e.g. Eckart 1960, Bildsten et al. 1996, Lee & Saio 1997)*
  - Strengths:
    - ✓ separable dynamics
    - ✓ tractable formalism / efficient for extensive seismic modelling
    - ✓ robust predictions for frequencies when compared to 2D adiabatic modes computation
  - Assumptions:
    - ✓ Strong stratification: $2\Omega \ll N$
    - ✓ Low frequency: $\omega \ll N$
      - ➢ Neglect the vertical (horizontal) component of the Coriolis acceleration (rotation vector)
    - ✓ adiabatic limit / Cowling approximation
    - ✓ spherical stars / uniform rotation *(e.g. Lee & Saio 1997)*
    - ✓ **no magnetic field**
  - Weaknesses:
    - ✓ Can't be used to deal with waves' excitation, damping, and instabilities *(e.g. Mathis et al. 2014)*
    - ✓ Not applicable in convective regions and in very rapidly rotating stars *(e.g. Ogilvie & Lin 2004, Dhouib et al. 2021a,b)*

Magnetic TAR

❖ **Magnetic Laplace tidal equation (Horizontal structure) :**

Assuming: $\omega_A \ll N$

$$\mathcal{L}^{\mathrm{magn}}_{\omega^{\mathrm{in}}\,m} = \omega^2 \partial_x \left[\frac{1}{\mathcal{A}} \frac{1-x^2}{D_{\mathrm{M}}} \partial_x \right] + m\omega^2 \partial_x \left(\frac{\nu_{\mathrm{M}} x}{\mathcal{A} D_{\mathrm{M}}} \right) - m^2 \frac{\omega^2}{\mathcal{A} D_{\mathrm{M}} (1-x^2)} + m^2 \frac{\omega^2}{\mathcal{A}^2} \frac{x}{D_{\mathrm{M}}} \partial_x \omega_{\mathrm{A}}^2$$

$$\mathcal{L}^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,m} \left[w^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,km}(r,x) \right] = -\Lambda^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,km}(r)\, w^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,km}(r,x)$$

Eigenvalues

Generalised Hough functions

$$x = \cos\theta$$
$$\mathcal{A} = \omega^2 - m^2 \omega_{\mathrm{A}}^2$$
$$\mathcal{B} = 2\left(\Omega\omega + m\omega_{\mathrm{A}}^2\right)$$
$$\nu_{\mathrm{M}} = \mathcal{B}/\mathcal{A}$$
$$D_{\mathrm{M}} = 1 - \nu_{\mathrm{M}}^2 x^2 - \left(1 - x^2\right) \frac{x}{\mathcal{A}} \partial_x \omega_{\mathrm{A}}^2$$

Magnetic spin parameter

Alfvén frequency

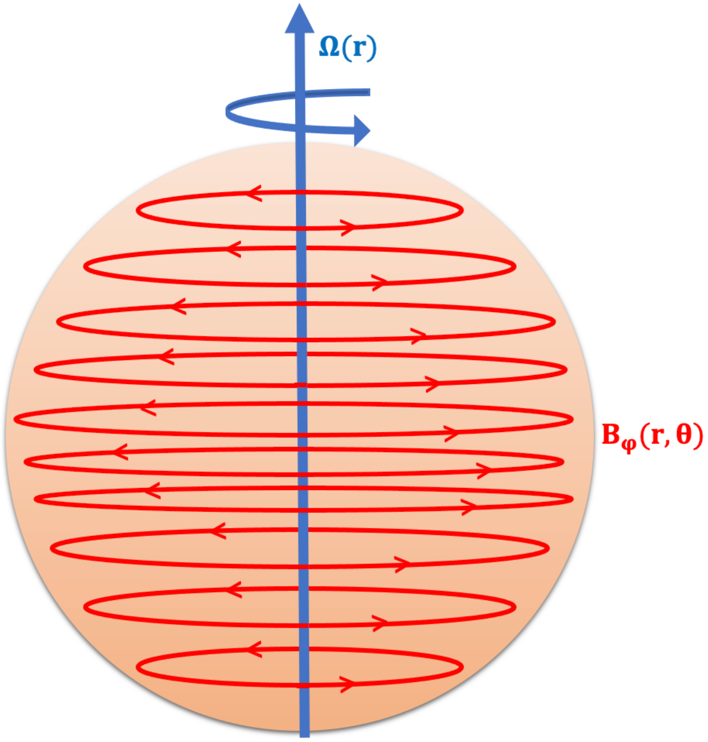
❖ **Dispersion relation + Radial quantification :**

$$\int_{r_1}^{r_2} \frac{N\sqrt{\Lambda^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,km}}}{r\omega}\, \mathrm{d}r = (n + 1/2)\pi$$

(2D JWKB approximation : Rapidly oscillating waves along the radial direction)

m: azimuthal order
k : latitudinal index ($k = \ell - |m|$)
n : radial order
ℓ : latitudinal order

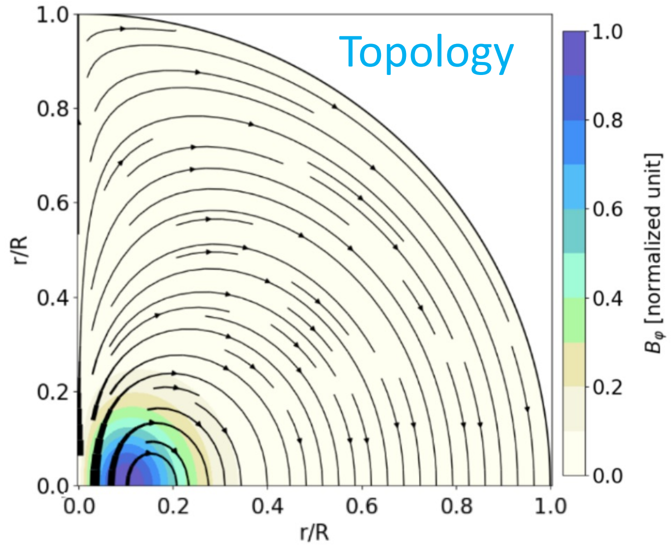
Setup:
- Differential rotation
- General axisymmetric toroidal field



$$B_\varphi(r,\theta) = \sqrt{\mu\rho_0}\, r \sin\theta\, \omega_{\mathrm{A}}(r,\theta)$$

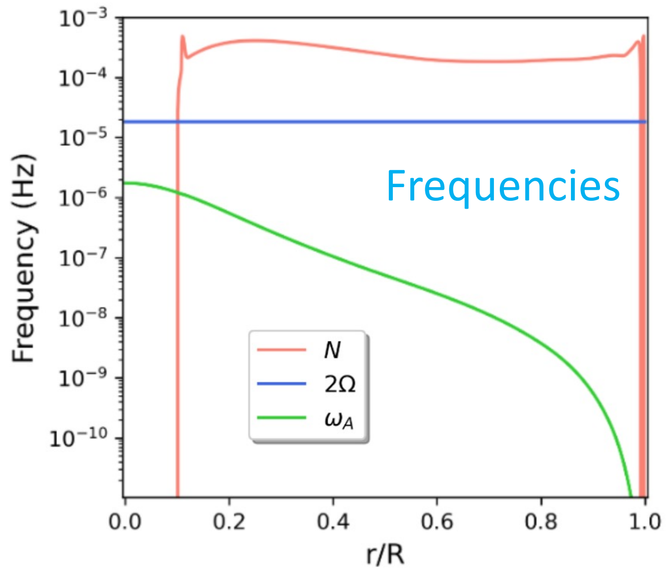
Dhouib et al. 2022

Case of equatorial toroidal field (1/2)

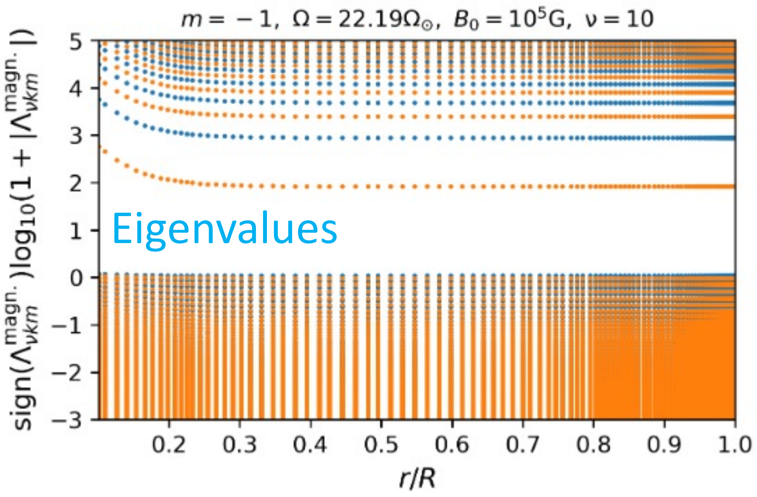


Initially fossil field

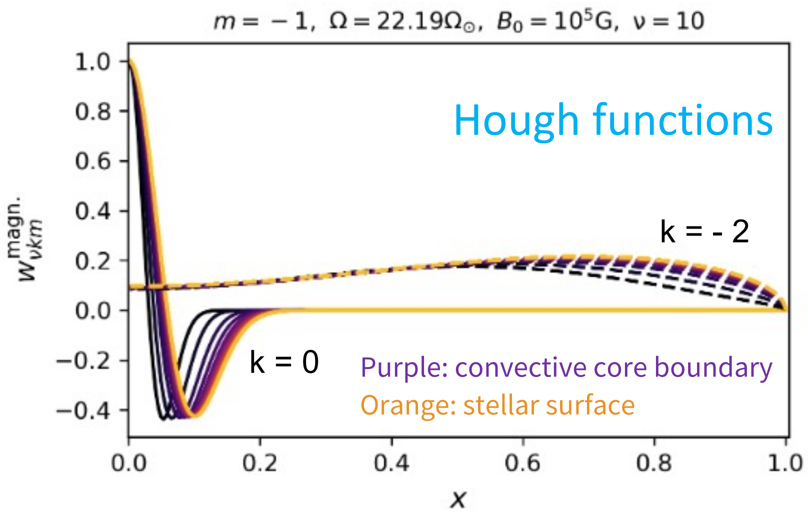
Sufficient radial differential rotation → efficient dynamo loop can take place



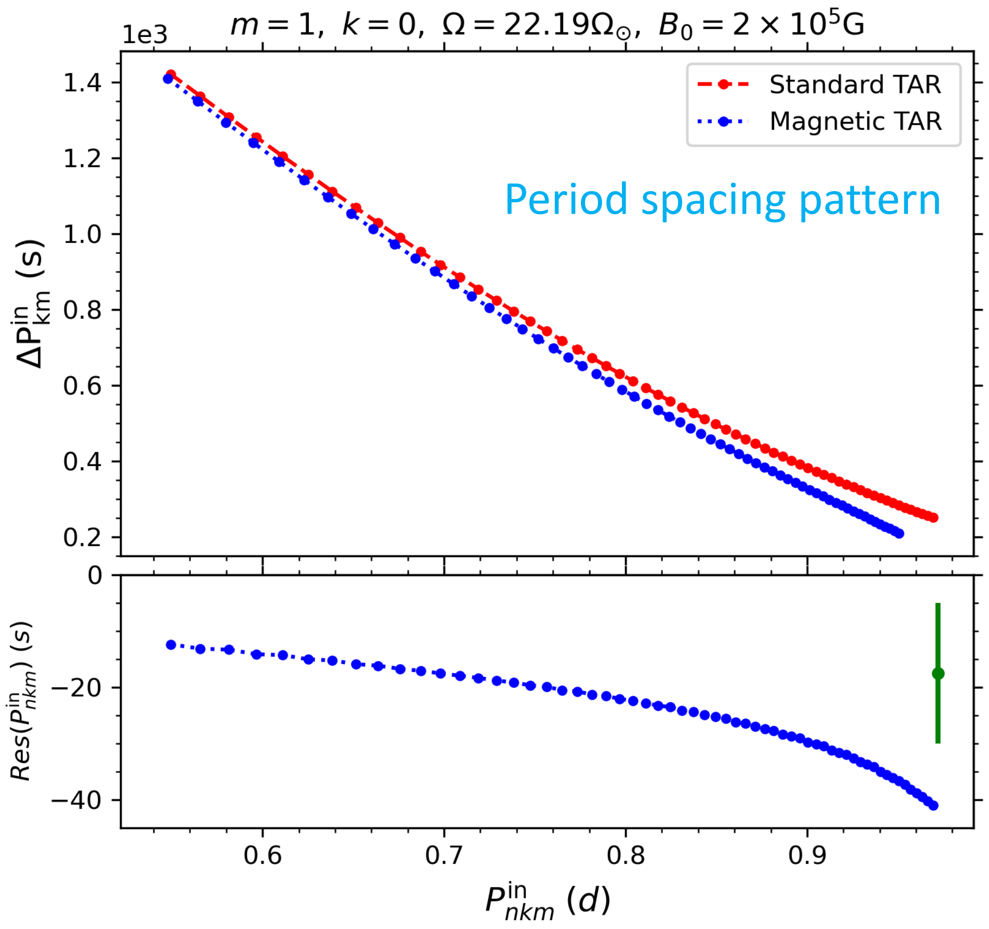
γ Dor model
$1.6\ M_\odot$
near ZAMS
$B_0 = 10^5$ G
$\Omega = 22.19\ \Omega_\odot$

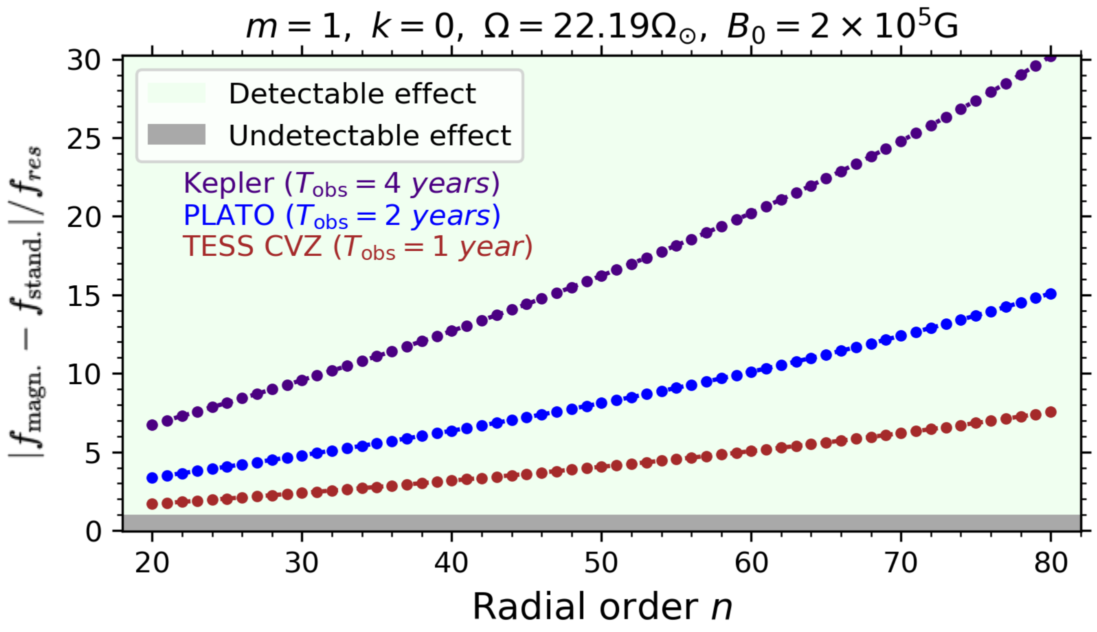


➢ **Strong radial dependence in the inner region of the star**



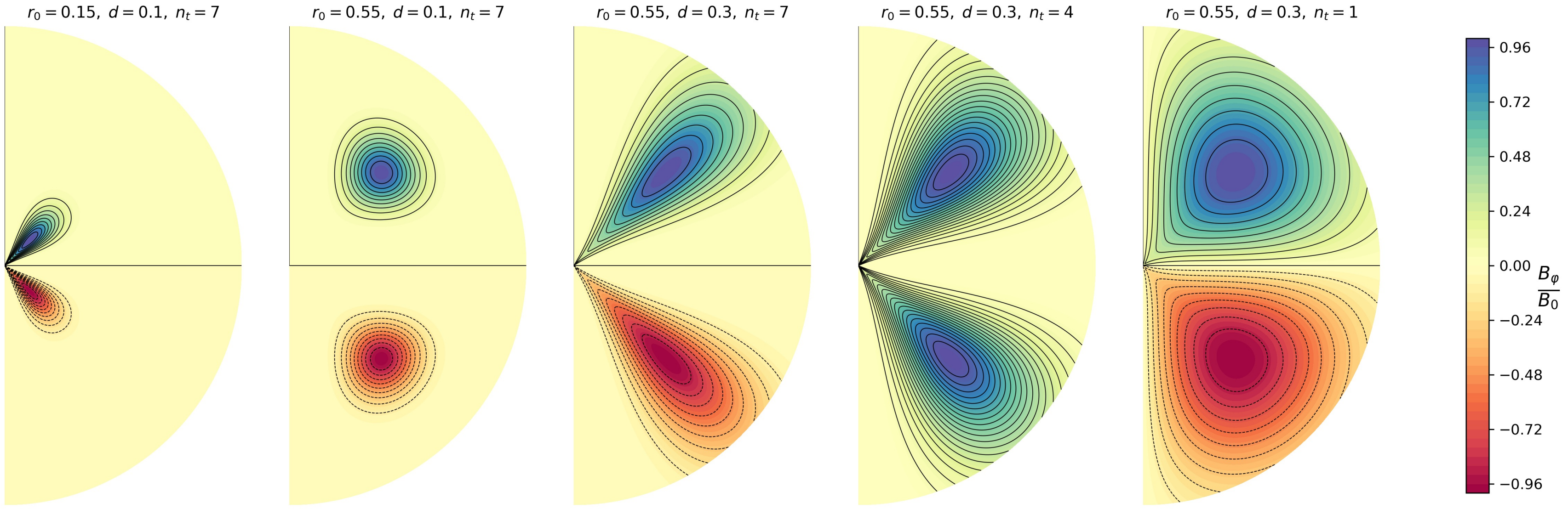
Case of equatorial toroidal field (2/2)





- ➢ A deep buried equatorial toroidal field can be detected
- ❖ Signature similar to rotation
- ❖ No sawtooth pattern seen with perturbative method
  (Prat et al. 2019, 2020, Van Beeck et al. 2020)
  - ✓ Possible cause:
    - absence of poloidal component?
    - asymptotic approach?

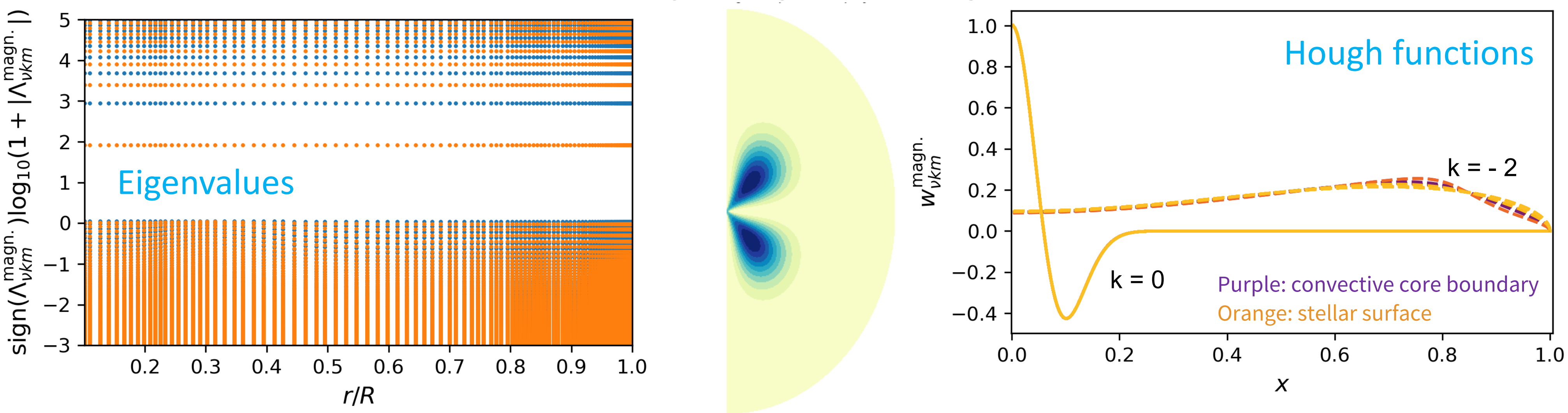
Case of hemispheric toroidal field (1/2)



$$B_\varphi(r,\theta) = B_0 \exp\left[-\frac{(r-r_0)^2}{2d^2}\right] \sin^{n_t}(2\theta)$$

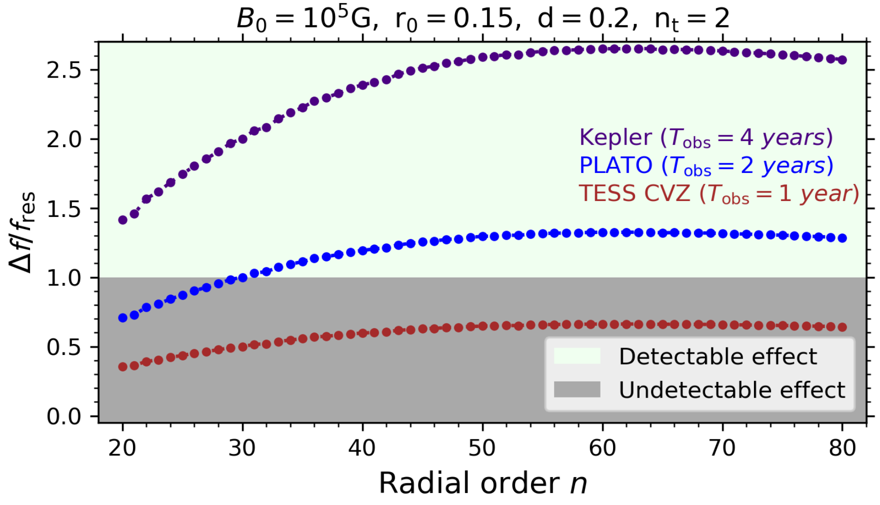
Depend on the history of the differential rotation *(Jouve et al. 2020)*

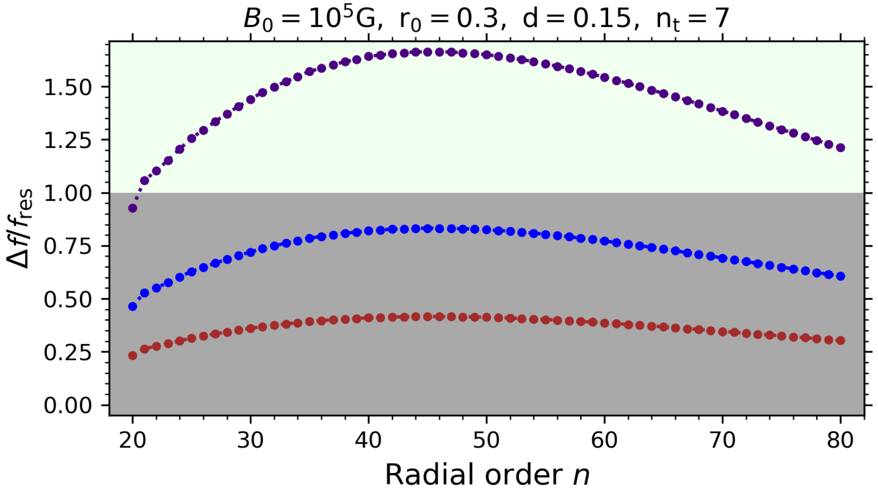
$m = -1,\ B_0 = 10^5\mathrm{G},\ r_0 = 0.2,\ d = 0.15,\ n_t = 2,\ \nu = 10$



➢ **Weak radius dependence**

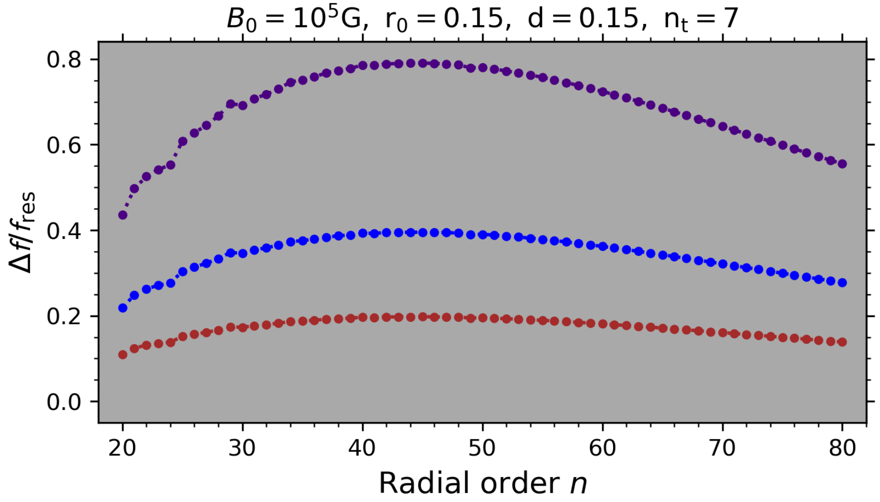
Case of hemispheric toroidal field (2/2)

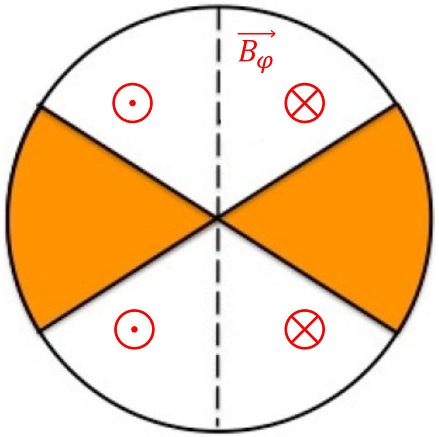




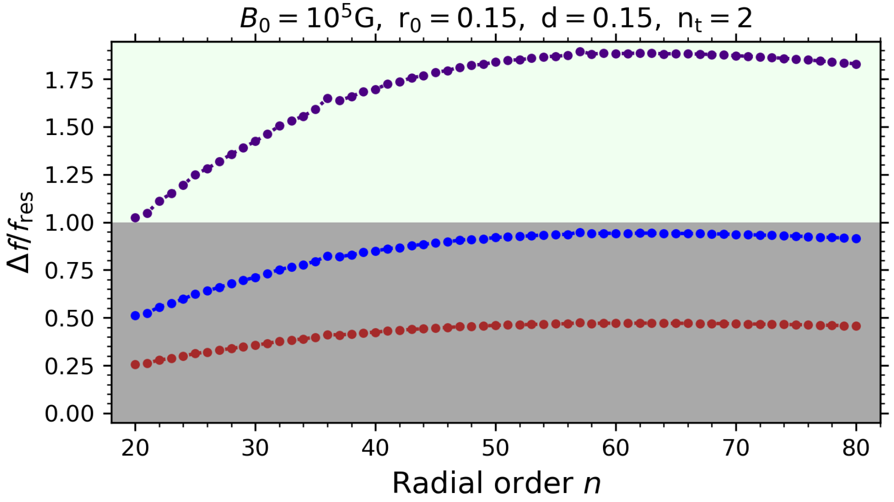
➢ Deep buried hemispheric fields difficult to detect

❖ MGI modes propagate in an equatorial belt





Mathis & de Brye 2012



Conclusions and perspectives

✓ Derivation of a **new generalisation of the TAR**

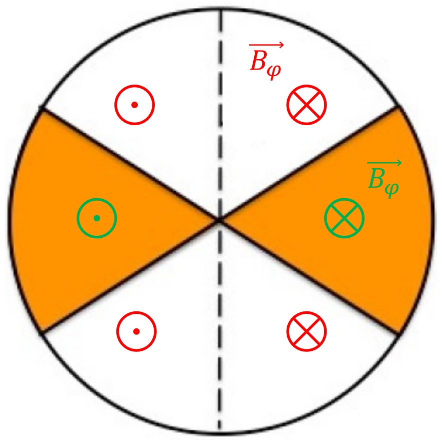
✓ Better **magneto-asteroseismic modelling**

✓ **Dependency** of the detectability of magnetic fields **on their configurations**

✓ **Possible detection** of equatorial fields

✓ Signature **similar to rotation**

✓ **Greater difficulty** in detecting fields located far from the equator



❖ Apply the magnetic TAR to **real stars** → improve **forward modelling**?

❖ Go beyond the asymptotic approach: **incorporate the magnetic TAR in GYRE**

❖ Go beyond the TAR: **3D MHD oscillation code**