The early universe as an open quantum system

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Outline

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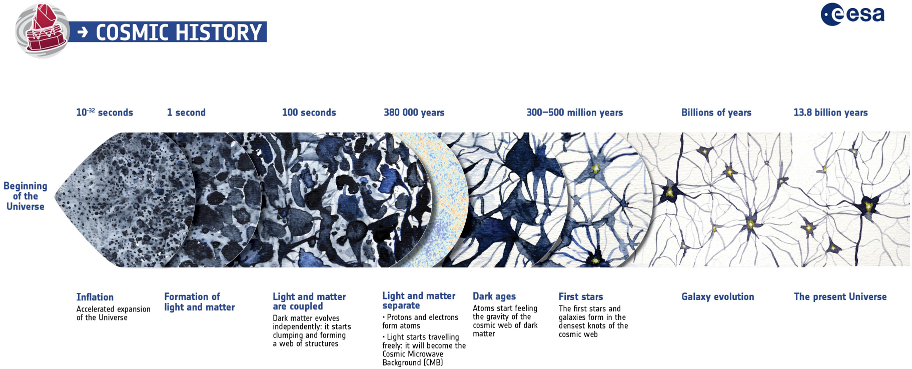
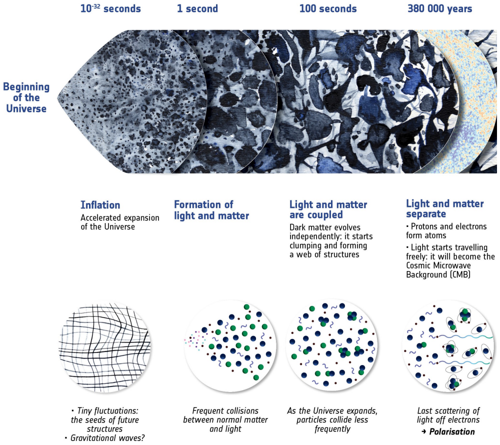
The standard model of cosmology



The standard model of cosmology



Quantum fluctuations of the primordial vacuum seed all the structures of the Universe $\Rightarrow$ Understanding this mechanism is crucial.

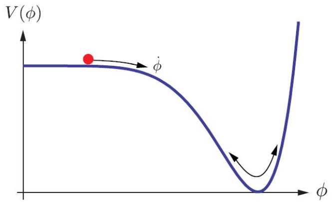
The generation of cosmic inhomogeneities

- Cosmology is a playground to test quantum mechanics in its most extreme regimes.
- In this description, the quantum state of the system plays a crucial role: it contains all the statistical information.
- For the simplest model of primordial cosmology, the quantum state is exactly known: it is a two-mode squeezed state

$$\left|2\mathrm{MSS}_{\vec{k}}\right\rangle = \sum_{n=0}^{\infty} \frac{e^{-2i(\theta_k+\varphi_k)}}{\cosh r_k}(-1)^n e^{-2in\theta_k} \tanh^n r_k \left|n_{\vec{k}}, n_{-\vec{k}}\right\rangle$$

Simplest attempt: slow-roll inflation

- A single scalar field, the inflaton ϕ, slowly rolling along its potential for a sufficiently long time:



$\Rightarrow$ exponentially expanding universe (quasi de-Sitter)

1. Enough to solve the Hot Big Bang puzzles; [Linde, 1982]
2. Quantum fluctuations are amplified;
3. Fit extremely well all cosmological observations.

Is it enough to describe a realistic early universe scenario?

Embedding inflation in a realistic setting

- Single-field slow roll inflation does not provide a complete picture.
  1. Inflation must connect with the late-time history of the universe.
  2. Inflation occurs at energy scale $< 10^{16}$ GeV. What came before?

  $\Rightarrow$ Even if the inflaton can be the leading ingredient of the matter content, it is not the only one: SM sector + UV sector.

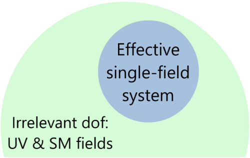
- Two-mode squeezed states only describe the linear self-interactions.
  1. GR non-linearities induce coupling betweeen scales.
  2. UV-modes can backreact on the IR-modes e.g.: [Brahma *et al., 2020*].

  $\Rightarrow$ Even if two-mode squeezed states contribute to the complete quantum description, they are not the final state.

What is the early universe made of?

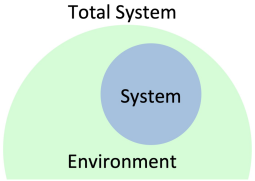
Three observations:

1. Single-field slow roll inflation provides an excellent fit of the data.
2. At some point, inflation must end: couple to SM fields.
3. UV-completions of inflation often introduce new degrees of freedom.



What are the quantum properties of the effective single-field system?

The early universe as an Open Quantum System (OQS)



- By integrating out the environment, the system dynamics becomes non-unitary.
- Cosmological perturbations are described by an OQS with dissipation and decoherence.
- They experience energy and information loss into the environment.

Effects of dissipation and decoherence

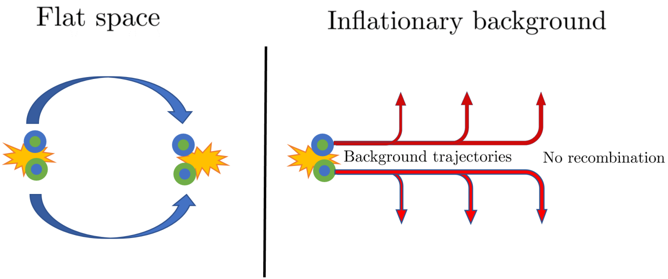
Two consequences:

1. Observables modified.
    - Observational window constrains parameter space of the environment.
2. Ability to measure genuine quantum features altered. [Joos & Zeh, 1985]
    - Decoherence often wash out desired quantum properties.

A peculiarity from curved-space physics:

- In the lab, states are initially prepared in entangled configurations.
- In inflationary context, entanglement continuously builds up.

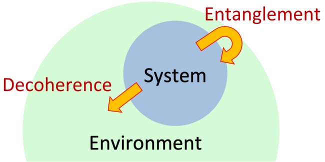
Pair creation in the early universe



Universe populated with highly entangled correlations

Coexistence of entanglement and decoherence

Entanglement builds up at the same time decoherence proceeds.



⇒ Cannot rely on the flat space intuitions of decoherence.

[Vennin & Martin, 2019]

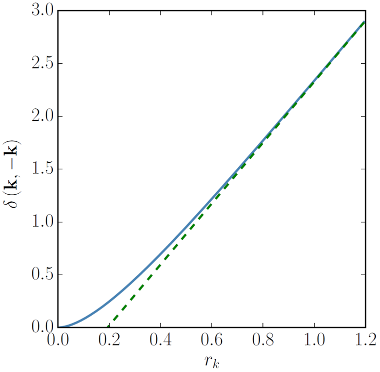


FIG. 1: Quantum discord $\delta(\boldsymbol{k}, -\boldsymbol{k})$ of cosmological scalar perturbations during inflation, as a function of the squeezing parameter r_k. The solid blue line is the result (44) while the dotted green line is the large squeezing expansion (45).

Decoherence in the cosmological context

- What is the leading effect between self-entanglement and decoherence?
- Decoherence can be seen as an issue, rendering difficult to prove the quantum origin of cosmic inhomogeneities.
- It can also be seen as a desired feature to explain the so-called quantum-to-classical transition.

$\Rightarrow$ **Clarifying the role played by decoherence** in the early universe would strengthen our understanding of primordial cosmology.

How does decoherence proceed in the early universe?

What are its effects?

How can we faithfully model it?

Outline

Two-field cosmology as a Cosmic OQS

In [Colas, Grain & Vennin, 2021], we studied two linearly coupled scalar fields in a homogeneous and isotropic background:

- *System*: one of the two fields (can be the inflaton, observable cosmological perturbations, ...) ;
- *Environment*: the other field (can be the Higgs field, unobservable cosmological perturbations, ...)
- *Interactions*: mimic linear order in perturbation theory: only allowed couplings are quadratic

$$\phi_1\phi_2; \qquad \pi_1\pi_2, ; \qquad \phi_1\pi_2; \qquad \pi_1\phi_2.$$

Implications of working at linear order

- No mode coupling: the state factorizes

$$|\Psi(t)\rangle = \prod_{\vec{k}} \left|\Psi_{\vec{k}}(t)\right\rangle$$

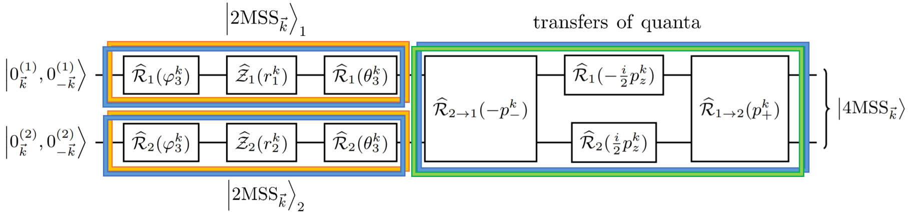
- Reduced phase-space underlied by symplectic structure:

  $\Rightarrow$ All constituents of the dynamics (Green's functions, evolution operator, canonical transformations, ...) are given in a representation of the Lie group $\mathrm{Sp}(4,\mathbb{R})$

Strategy:

1. Study $\mathrm{Sp}(4,\mathbb{R})$;
2. Find evolution operator $\widehat{\mathcal{U}}_{\vec{k}}(t,t_0)$;
3. Find final quantum state $\left|\Psi_{\vec{k}}(t)\right\rangle = \widehat{\mathcal{U}}_{\vec{k}}(t,t_0)\left|\Psi_{\vec{k}}(t_0)\right\rangle$;
4. Trace-out the environment $\widehat{\rho}_{\mathrm{red}}(t) = \mathrm{Tr}_E \left|\Psi_{\vec{k}}(t)\right\rangle\left\langle\Psi_{\vec{k}}(t)\right|$.

Four-mode squeezed states



which only depends on three elementary operations

$$\overbrace{\widehat{\mathcal{R}}_i(\theta^k) \equiv e^{i\theta^k\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}_{i,\vec{k}} + \widehat{a}^\dagger_{i,-\vec{k}}\widehat{a}_{i,-\vec{k}} + 1\right)}}^{\text{phase shift}}, \qquad \overbrace{\widehat{\mathcal{Z}}_i(r^k) \equiv e^{r^k\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}^\dagger_{i,-\vec{k}} - \widehat{a}_{i,\vec{k}}\widehat{a}_{i,-\vec{k}}\right)}}^{\text{pair creation}}$$

$$\underbrace{\widehat{\mathcal{R}}_{i\to j}(p^k) \equiv e^{ip^k\left(\widehat{a}^\dagger_{j,\vec{k}}\widehat{a}_{i,\vec{k}} + \widehat{a}^\dagger_{j,-\vec{k}}\widehat{a}_{i,-\vec{k}}\right)}}_{\text{transfers of quanta}}$$

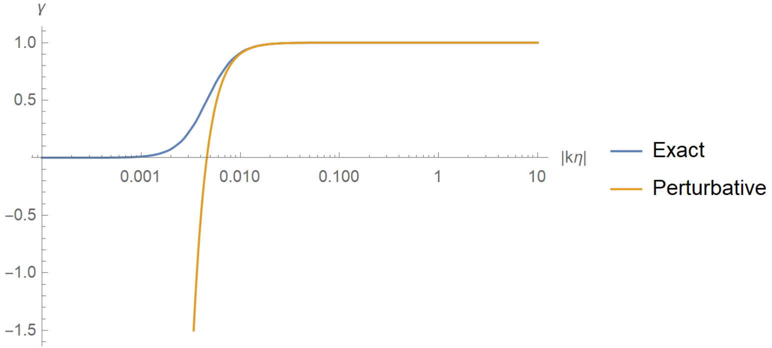
Outline

Assessing models of decoherence

- One can model the effects of the environment by using the cosmological master equation [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Hollowood & McDonald, 2017], [Martin & Vennin, 2018], [Brahma, Berera & Calderón-Figueroa, 2021], etc.
- Yet, it relies on assumptions: small coupling with the environment, following **Born approximation**.
- We propose to assess this approximation scheme on an exactly solvable model.
  - Thanks to the tools discussed previously, we have analytic control on the decoherence of the system.
  - We can then compare these exact results with the one obtained from the cosmological master equation.

Perturbative regime

- We compute the **power spectra** and the **purity**:
  1. from the master equation;
  2. by perturbing the exact results.
- Results **exactly match** $\Rightarrow$ master equation is a valid perturbative method.
- Perturbative regime depends on $\lambda^2 a^2$ $\Rightarrow$ breaks down at late-time.



Late-time resummation

- Cosmological master equation known for its ability to resum late-time secular effets [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015].
- Want to exhibit this feature and to understand **what is being resumed**. In particular, can we recover the purity of the state?
- By decomposing the light sector in **growing and decaying modes**, capture the leading late-time contribution [Brahma, Berera & Calderón-Figueroa, 2021]

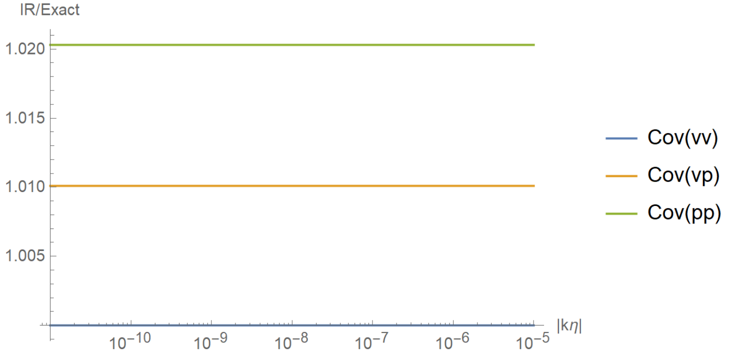
$$\mathrm{Cov}_{11,k}^{(vv)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}|v_{1,\boldsymbol{k}}(\eta)|^2$$
$$\mathrm{Cov}_{11,k}^{(pp)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}|p_{1,\boldsymbol{k}}(\eta)|^2$$
$$\mathrm{Cov}_{11,k}^{(vp)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}\,\mathrm{Re}\left\{v_{1,\boldsymbol{k}}(\eta)p^*_{1,\boldsymbol{k}}(\eta)\right\}.$$

Comparison with exact results at late-time

$$\mathrm{Cov}_{11,k}^{(vv)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}\left|v_{1,\boldsymbol{k}}(\eta)\right|^2$$

$$\mathrm{Cov}_{11,k}^{(pp)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}\left|p_{1,\boldsymbol{k}}(\eta)\right|^2$$

$$\mathrm{Cov}_{11,k}^{(vp)\ \mathrm{IR}}(\eta) = e^{-\frac{1}{\nu_1}\frac{\lambda^4}{M^2}\ln(-k\eta)}\,\mathrm{Re}\left\{v_{1,\boldsymbol{k}}(\eta)p^*_{1,\boldsymbol{k}}(\eta)\right\}.$$



Summary

In general:

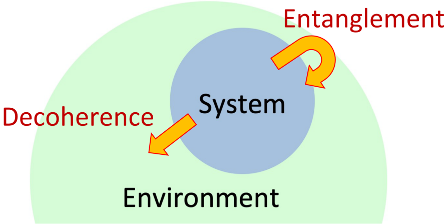
1. Cosmology is a playground to test quantum mechanics in its most extreme regimes.
2. It requires to accurately describe the quantum state of the universe: we develop an Open Quantum System approach.
3. In this setting, the dynamics is non-unitary: it experiences dissipation and decoherence.

On our work:

1. We derived the quantum states of two-field cosmology: the four-mode squeezed states.
2. We benchmarked an effective approach of decoherence: the cosmological master equation.

Outlook

If the system exhibits Bell's inequality violation, can this feature survive environmental decoherence?



Can we observationally prove the quantum origin of cosmic inhomogeneities?

Thank you for your attention !

References

T.Colas, J. Grain, V. Vennin,
Four-mode squeezed states: two-field quantum systems and the symplectic group $\mathrm{Sp}(4, \mathbb{R})$
arXiv:2104.14942

T.Colas, J. Grain, V. Vennin,
Curved-space Caldeira-Leggett model as benchmark for cosmological master equation
arXiv:21XX.XXXXX

Outline

Cosmological Perturbation Theory

- Expand the metric and the field around the background FLRW solution.

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_i \\ -B_i & 2[\psi\delta_{ij} - E_{ij}], \end{pmatrix} \text{ and } \varphi(\eta, \vec{x}) = \bar{\varphi}(\eta) + \delta\varphi(\eta, \vec{x})$$

- Need to formulate the dynamics in a gauge invariant way. There is only one propagating gauge invariant degree of freedom in the scalar sector. Can choose it to be the Mukhanov-Sasaki variable:

$$v \propto \phi, \psi, B, E, \delta\varphi$$

- Perturb the Einstein-Hilbert action. At linear order

$$S = \frac{1}{2}\int \mathrm{d}^4 x \left[(v')^2 - (v_{,i})^2 + \frac{z''}{z} v^2\right],$$

$\Rightarrow$ a free scalar field with a time dependent mass $m_{\text{eff}}^2 = -z''/z$.

- *Quantization*: fields are promoted to quantum operators and decomposed in their Fourier representation. Introduce $\widehat{a}_{\vec{k}}$ and $\widehat{a}_{\vec{k}}^{\dagger}$:

$$\widehat{v}_{\vec{k}} = \frac{1}{\sqrt{2k}}\left(\widehat{a}_{\vec{k}} + \widehat{a}^{\dagger}_{-\vec{k}}\right) \text{ and } \widehat{\pi}_{\vec{k}} = -i\sqrt{\frac{k}{2}}\left(\widehat{a}_{\vec{k}} - \widehat{a}^{\dagger}_{-\vec{k}}\right),$$

- The evolution operator writes [Albrecht *et al.*, 1994]

$$\widehat{\mathcal{U}}(\eta,\eta_0) = \mathcal{S}\left(r_k,\theta_k\right)\mathcal{R}\left(\varphi_k\right),$$

with

$$\begin{aligned} \mathcal{R}\left(\varphi_k\right) &= \exp\left(i\varphi_k\left[\widehat{a}^{\dagger}_{\vec{k}}\widehat{a}_{\vec{k}} + \widehat{a}^{\dagger}_{-\vec{k}}\widehat{a}_{-\vec{k}} + 1\right]\right) \\ \mathcal{S}\left(r_k,\theta_k\right) &= \exp\left(r_k\left[e^{2i\theta_k}\widehat{a}^{\dagger}_{\vec{k}}\widehat{a}^{\dagger}_{-\vec{k}} - \text{h.c.}\right]\right). \end{aligned}$$

- Starting from an initial vacuum, the quantum state of the system at any time η reads

$$\left|\emptyset(\eta)\right\rangle = \widehat{\mathcal{U}}(\eta,\eta_0)\left|\emptyset(\eta_0)\right\rangle.$$

Contact with observations

- From the quantum state, obtain the correlators, eg.:

$$P_{\vec{k},\vec{q}}(\eta) = \langle \emptyset(\eta) | \, \widehat{v}_{\vec{k}} \widehat{v}_{\vec{q}}^{\dagger} \, | \emptyset(\eta) \rangle \, .$$

$\Rightarrow$ predicts a nearly scale invariant power spectrum, consistent with observations from *Planck*.

- Highlight 1: amplification of quantum fluctuations explains the statistics of the CMB temperature anisotropies.
- Highlight 2: the quantum state is important as it contains the information we need to make observational predictions.

Outline

Classical dynamics

- Linear dynamics in the Hamiltonian framework:

$$H = \int_{\mathbb{R}^{3+}} \mathrm{d}^3 k \, \boldsymbol{z}_{\vec{k}}^{\dagger} \boldsymbol{H}_k \boldsymbol{z}_{\vec{k}}.$$

- Equations of motion:

$$\dot{\boldsymbol{z}}_{\vec{k}} = \left(\Omega \boldsymbol{H}_k\right) \boldsymbol{z}_{\vec{k}}, \qquad \Omega \boldsymbol{H}_k \in \mathfrak{sp}(4, \mathbb{R}).$$

- Green's matrix formalism:

$$\dot{\boldsymbol{G}}_k = \left(\Omega \boldsymbol{H}_k\right) \boldsymbol{G}_k + \mathsf{Id}_4 \delta(t - t_0),$$

and

$$\boldsymbol{z}_{\vec{k}}(t) = \boldsymbol{G}_k(t, t_0) \boldsymbol{z}_{\vec{k}}(t_0), \qquad \boldsymbol{G}_k(t, t_0) \in \mathrm{Sp}(4, \mathbb{R})$$

What are $\mathrm{Sp}(4,\mathbb{R})$ and $\mathfrak{sp}(4,\mathbb{R})$?

- Symplectic group and algebra:

$$
\begin{aligned}
\mathrm{Sp}(4,\mathbb{R}) &= \{M \in \mathcal{M}_4(\mathbb{R}) : M^T \Omega M = \Omega\}. \\
\mathfrak{sp}(4,\mathbb{R}) &= \{X \in \mathcal{M}_4(\mathbb{R}) : \Omega X + X^T \Omega = 0\}.
\end{aligned}
$$

with

$$
\Omega = \begin{pmatrix} 0 & \mathrm{Id}_2 \\ -\mathrm{Id}_2 & 0 \end{pmatrix}
$$

- Can be decomposed into [Hasebe, 2019]:

$$
\mathrm{Sp}(4,\mathbb{R})/H^{2,2} \cong \mathrm{Sp}(2,\mathbb{R}) \times \mathrm{Sp}(2,\mathbb{R}).
$$

Gaining intuition through the quantum representation

- Introduce creation $\widehat{a}^{\dagger}_{i_{\pm\vec{k}}}$ and annihilation $\widehat{a}_{i_{\pm\vec{k}}}$ operators.
- $\mathrm{Sp}(4,\mathbb{R})$ decomposes into
  - Two single-field copies: $\mathrm{Sp}(2,\mathbb{R}) \times \mathrm{Sp}(2,\mathbb{R})$

$$
\begin{array}{ccc}
\widehat{a}^{\dagger}_{1_{\vec{k}}}\widehat{a}^{\dagger}_{1_{-\vec{k}}}, & \widehat{a}_{1_{\vec{k}}}\widehat{a}_{1_{-\vec{k}}}, & \widehat{a}^{\dagger}_{1_{\vec{k}}}\widehat{a}_{1_{\vec{k}}} + \widehat{a}^{\dagger}_{1_{-\vec{k}}}\widehat{a}_{1_{-\vec{k}}} + 1; \\
\widehat{a}^{\dagger}_{2_{\vec{k}}}\widehat{a}^{\dagger}_{2_{-\vec{k}}}, & \widehat{a}_{2_{\vec{k}}}\widehat{a}_{2_{-\vec{k}}}, & \widehat{a}^{\dagger}_{2_{\vec{k}}}\widehat{a}_{2_{\vec{k}}} + \widehat{a}^{\dagger}_{2_{-\vec{k}}}\widehat{a}_{2_{-\vec{k}}} + 1;
\end{array}
$$

  - Interaction between the two sectors:

$$
\begin{array}{cc}
\widehat{a}^{\dagger}_{1_{\vec{k}}}\widehat{a}_{2_{\vec{k}}} + \widehat{a}^{\dagger}_{1_{-\vec{k}}}\widehat{a}_{2_{-\vec{k}}}, & \widehat{a}^{\dagger}_{2_{\vec{k}}}\widehat{a}_{1_{\vec{k}}} + \widehat{a}^{\dagger}_{2_{-\vec{k}}}\widehat{a}_{1_{-\vec{k}}}; \\
\widehat{a}^{\dagger}_{1_{\vec{k}}}\widehat{a}^{\dagger}_{2_{-\vec{k}}} + \widehat{a}^{\dagger}_{2_{\vec{k}}}\widehat{a}^{\dagger}_{1_{-\vec{k}}}, & \widehat{a}_{1_{\vec{k}}}\widehat{a}_{2_{-\vec{k}}} + \widehat{a}_{2_{\vec{k}}}\widehat{a}_{1_{-\vec{k}}}.
\end{array}
$$

Why do we matter?

- Allows to characterize the dynamics:

$$\boldsymbol{G}_k(\eta, \eta_0) \in \mathrm{Sp}(4, \mathbb{R}) \quad \Rightarrow \quad \boldsymbol{G}_k(t, t_0) = R(\boldsymbol{\theta}) Z(\mathbf{d}) R(\boldsymbol{\varphi})$$

- Quantum analogue: the evolution operator

$$\widehat{\mathcal{U}}(\eta, \eta_0) = \widehat{\mathcal{R}}(\boldsymbol{\theta}) \widehat{\mathcal{Z}}(\boldsymbol{r}) \widehat{\mathcal{R}}(\boldsymbol{\varphi}).$$

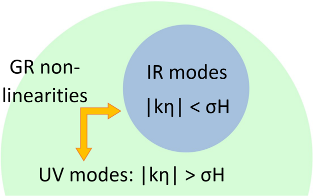
- Obtain the quantum state of the system at any time

$$|\emptyset(\eta)\rangle = \widehat{\mathcal{U}}(\eta, \eta_0) \, |\emptyset(\eta_0)\rangle \, ,$$

from which we get all the information on the system.

Outline

The OpenEFT formalism



In [Brahma *et al.*, 2020], the leading cubic contribution is

$$H_{\mathrm{int}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^3 x \varepsilon_H^2 a \zeta^2 \partial^2 \zeta$$

- UV modes backreact on the IR dynamics.
- They induce decoherence of the IR sector.