Convective instability in core-collapse supernovae

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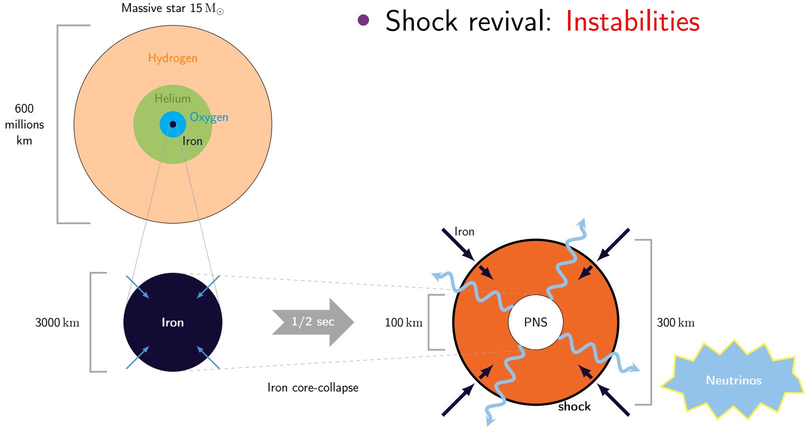
CEA/AIM

The collapse and the stalled shock

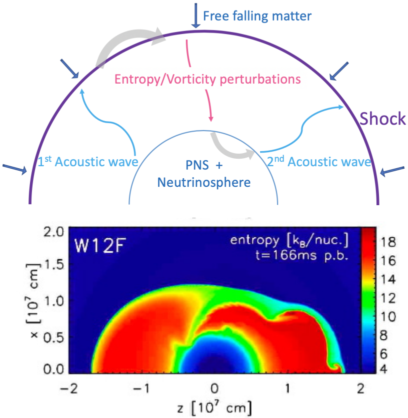
Massive stars end-of-life (8 to 40 $M_\odot$) :



- Collapse: 10^3km$\rightarrow$ 10^2km
- Shock wave created $\rightarrow$ stationnary
- Shock revival: Instabilities

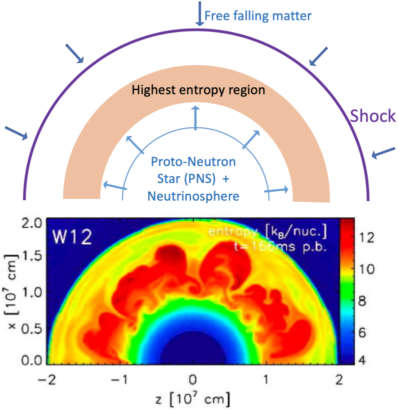
Standing Accretion Shock Instability (SASI) and Neutrino-driven convection

SASI



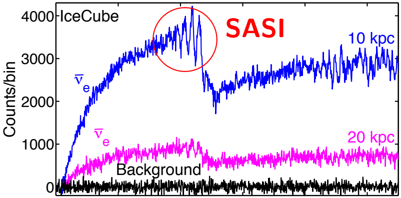
Foglizzo+2006

Neutrino-driven convection



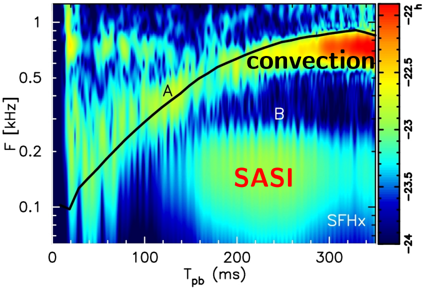
Expected observations

Neutrinos (27 $M_\odot$)



Tamborra+2013

Gravitational waves (15 $M_\odot$, 10kpc)



Kuroda+2016

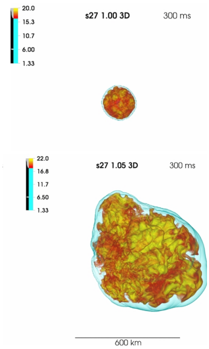
PhD

- Which instability occurs during the stationary phase?
- Which instability dominates?

The necessity of analytic criteria

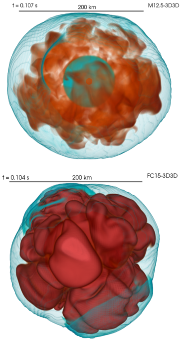
A large parameter space:

Heating function:



Couch & O'Connor 2014

Progenitor mass:



Vartanyan+2021

A simplified model to understand the neutrino driven convection

Methods

Physical hypothesis

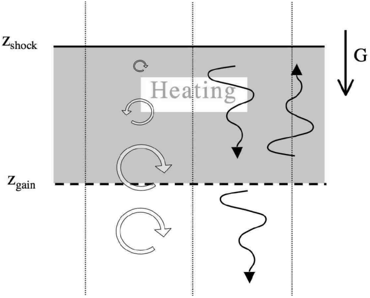
↕

Comparison to toy models (Planar/Spherical)

↙ ↘

Analytically | Numerically

A planar toy model

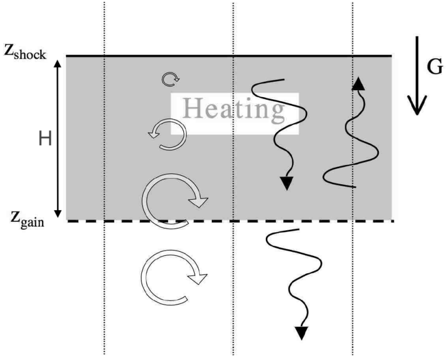


Foglizzo+2006

The stability criterion for convection

Physical hypothesis: The system is stable when $\frac{H}{|v|} \lesssim N^{-1}$

- $\tau_{adv} = \frac{H}{|v|}$,
- $\tau_{buoy} = \frac{1}{N}$, (N, the Brunt-Väisälä frequency)
- H, the length scale



(Foglizzo+2006, Ott+2013, Nakamura+2014, Takiwaki+2014, Glas+2019)

The stability criterion for convection

Stability criterion : $\chi_{crit} = \frac{\tau_{adv}}{\tau_{buoy}} \sim 3$

- $\tau_{adv} = \frac{H}{|v|}$,
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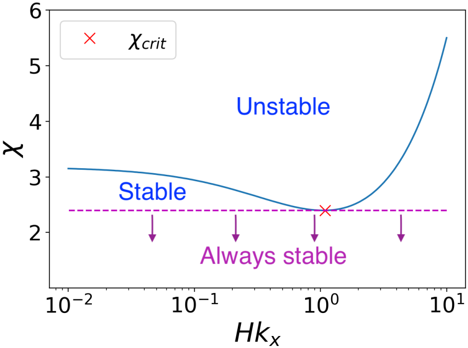
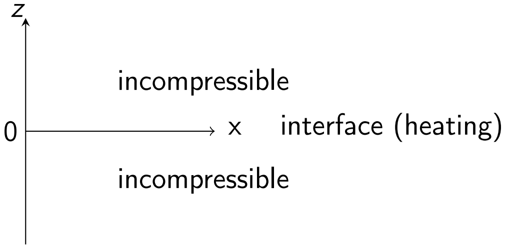


(Foglizzo+2006, Ott+2013, Nakamura+2014, Takiwaki+2014, Glas+2019)

Intuitive counter-example

$$\chi = \underbrace{\sqrt{\frac{\gamma - 1}{\gamma}\, g\, \frac{\Delta S}{H}}}_{N} \times \frac{H}{v} \propto \sqrt{H}$$

- 2 incompressible fluids and density discontinuity: $\omega^2 = -A_t k_x g$ with $A_t = \frac{\rho_d - \rho_u}{\rho_d + \rho_u}$.



Definition problem

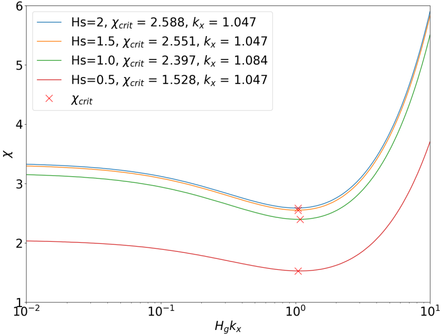
Stability criterion: $$\chi = \frac{\tau_{adv}}{\tau_{buoy}} = \underbrace{\sqrt{\frac{\gamma - 1}{\gamma} g \frac{\Delta S}{H}}}_{N} \times \frac{H}{v}$$

- Advection timescale τ_{adv} defined over which region?
- Buoyancy time τ_{buoy} defined at which altitude and which horizontal wavelength?

Length scales

Stability criterion : $\chi_{crit} = \frac{\tau_{adv}}{\tau_{buoy}} \sim 3$

- $\tau_{adv} = \frac{H}{|v|}$
- $\tau_{buoy} = \frac{1}{N}$
- H_g, length scale of the non-zero gravity g
- H_s, entropy length scale



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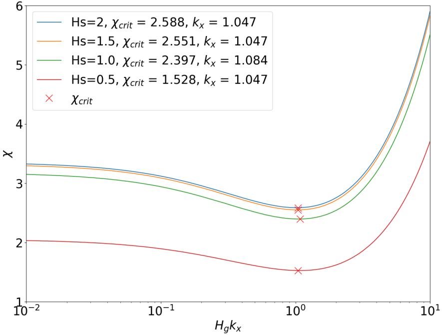


Need to define a more global new criterion

Timescale definition

Advection timescale definition: $\tau_{adv} = \dfrac{H}{|v|} \longrightarrow \dfrac{H_{pert.}}{|v|}$

H_{pert}, vertical extension of perturbations

(incompressible case : $H_{pert} = \dfrac{1}{k_x}$)

Timescale definition

Advection timescale definition: $\tau_{adv} = \frac{H}{|v|} \longrightarrow \frac{H_{pert.}}{|v|}$

H_{pert}, vertical extension of perturbations

(incompressible case : $H_{pert} = \frac{1}{k_x}$)

Buoyancy timescale definition: $\tau_{buoy} = \frac{1}{N} \longrightarrow \frac{1}{\omega}$

Small wavelength, large k_x $\longrightarrow$ Valid for all k_x

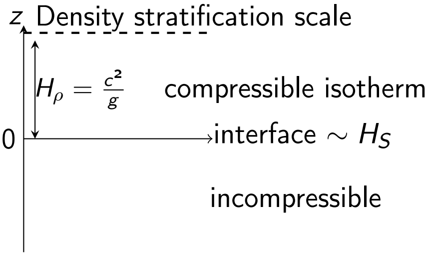
New criterion, analytical approach

Physical hypothesis:

- Adiabatic perturbations
- The system is stable when

$$\frac{H_{pert.}}{|v|} \lesssim \omega^{-1}$$

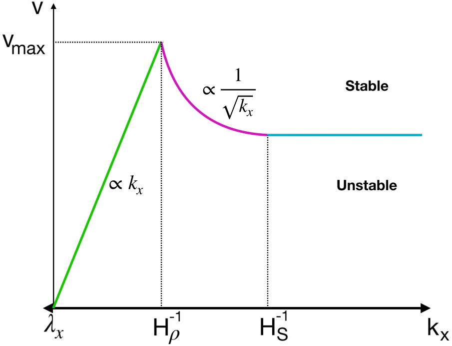
ω, hydrostatic growth rate



New criterion, analytical expectations

Several length scales :

- H_S, the size of the most buoyant region
- $H_\rho = \frac{c_s^2}{g}$, the density scale-height
- H_g, the size of the gain region

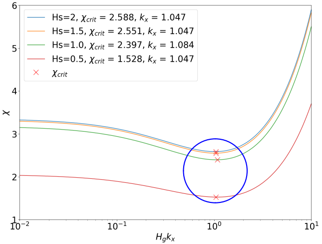


Analytical expectation:

$$\chi_{crit} \propto \sqrt{\frac{H_s}{H_\rho}}$$

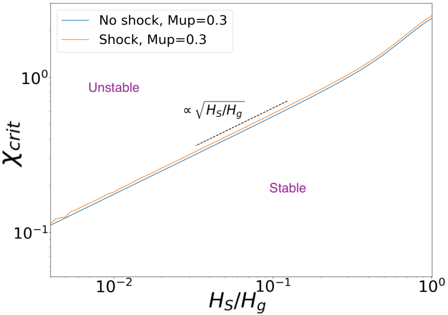
$$k_{x,crit} \propto \frac{1}{H_\rho}$$

Length scales, numerical approach

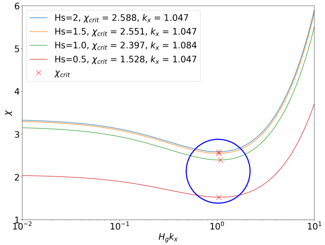


$$\chi_{crit} \propto \sqrt{\frac{H_s}{H_\rho}}$$

Numerical result:



Length scales, numerical approach



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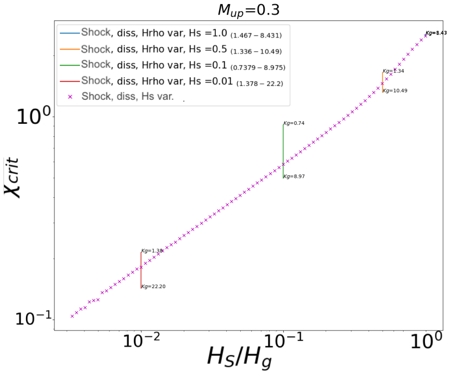
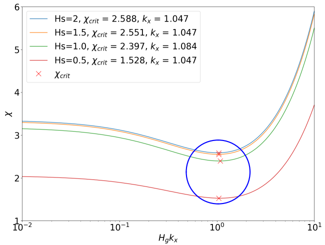
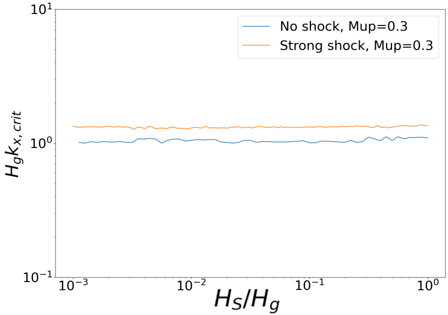


Length scales, numerical approach



Numerical result:

$k_{x,crit}$ independent of H_S



The new definition of the threshold

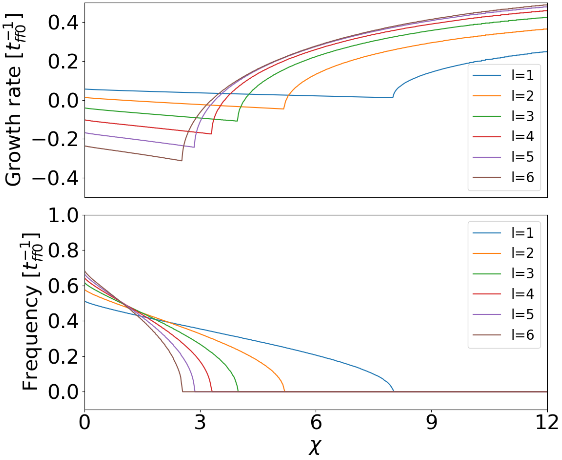
Old definition: $\chi_{06} = \int_{z_{gain}}^{z_{sh}} \frac{N(z)}{|v_z|} \mathrm{dz}$,

with $N^2 = \frac{\gamma - 1}{\gamma} g \nabla S$, the Brunt-Väisälä frequency.

New definition: $\psi = \omega \int_{z_{gain}}^{z_{gain}+H_{pert.}} \frac{dz}{|v_z|}$

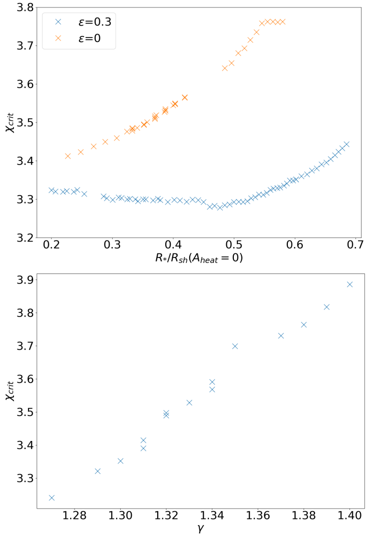
with ω the hydrostatic growth rate of the instability and H_{pert}, vertical extension of perturbations.

The convective instability in the supernova case



Yamasaki & Yamada 2006, Fernandez+2014

Constraints on χ_{crit}



- χ_{crit} increases with the shock radius and the adiabatic index
- χ_{crit} decreases with the dissociation

Conclusion

- Study of χ
- Definition of a new criterion ψ, more global
- Validation of this criterion in the planar case
- Small variation of χ_{crit} in the SN case: $\chi_{crit} \in [3, 4]$
- Standing Accretion Shock Instability (SASI) dominates for small heating rates

Next steps:

- Validation of the criterion ψ in the spherical case
- Influence of the rotation